

# Achievable Rates and Optimal Resource Allocation for Imperfectly-Known Fading Relay Channels

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# Introduction

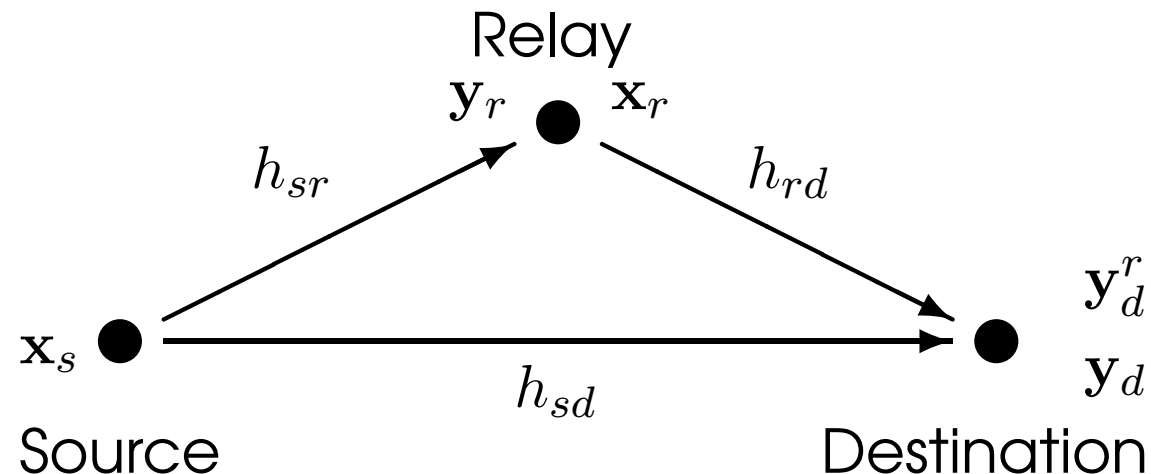
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- It is assumed that communication starts with the network training phase in which the receivers estimate the fading coefficients of their respective channels
- In the data transmission phase, amplify-and-forward and decode-and-forward relaying schemes with different degrees of cooperation are considered, and the corresponding achievable rate expressions are obtained
- Three resource allocation problems are addressed: 1) power allocation between data and training symbols; 2) time/bandwidth allocation to the relay; 3) power allocation between the source and relay in the presence of total power constraints.

# Channel Model

- We consider the three-node relay network which consists of a source, destination, and a relay node.
- Source-destination, source-relay, and relay-destination channels are modeled as Rayleigh block-fading channels that are constant for a block of  $m$  symbols.
- Overall, the source and relay are subject to the following power constraints in one block:

$$\|\mathbf{x}_{s,t}\|^2 + E\{\|\mathbf{x}_s\|^2\} \leq mP_s, \quad \|\mathbf{x}_{r,t}\|^2 + E\{\|\mathbf{x}_r\|^2\} \leq mP_r.$$



# Network Training

- Two symbol periods for Training. One estimates S-R ( $h_{sr}$ ), S-D ( $h_{sd}$ ). One for R-D ( $h_{rd}$ )
- it is assumed that the receivers employ minimum mean-square-error (MMSE) estimation.

- $$\hat{h}_{sr} = \frac{\sigma_{sr}^2 \sqrt{\delta_s m P_s}}{\sigma_{sr}^2 \delta_s m P_s + N_0} y_{r,t}$$

$$y_{r,t} \sim \mathcal{CN}(0, \sigma_{sr}^2 \delta_s m P_s + N_0).$$

We denote by  $\tilde{h}_{sr}$  the estimate error which is a zero-mean complex Gaussian random variable with variance

$$\text{var}(\tilde{h}_{sr}) = \frac{\sigma_{sr}^2 N_0}{\sigma_{sr}^2 \delta_s m P_s + N_0}.$$

For the fading coefficients  $h_{sd}$  and  $h_{rd}$ , we have similar results.



# Data Transmission

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- $m - 2$  symbols periods are left for data transmission
- Relay works under half-duplex constraint
- We introduce the relay transmission parameter  $\alpha$  and assume that  $\alpha(m - 2)$  symbols are allocated for relay transmission
- Transmission protocols can be classified into two categories by whether or not the source and relay simultaneously transmits information: non-overlapped and overlapped transmission.

In non-overlapped transmission, source transmits over a duration of  $(1 - \alpha)(m - 2)$  symbols and becomes silent as the relay transmits. On the other hand, in overlapped transmission, source transmits all the time and sends  $m - 2$  symbols in each block



# Non-overlapped transmission

- Amplify and Forward & Repetition Decode and Forward  
source and relay should be allocated equal time slots in the cooperation phase.  
we initially have direct transmission from the source to the destination over a duration of  $(1 - 2\alpha)(m - 2)$  symbols. In the Subsequent  $2\alpha(m - 2)$  symbols, cooperative transmission happens. Here  $0 < \alpha \leq 1/2$
- Parallel Decode and Forward channel coding,  
the source and relay do not have to be allocated the same duration in the cooperation phase.  
source transmits over a duration of  $(1 - \alpha)(m - 2)$  symbols while the relay transmits in the remaining duration of  $\alpha(m - 2)$  symbols. Here  $0 < \alpha \leq 1$



# *Overlapped transmission*

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- In AF and repetition DF, similarly as in the non-overlapped model, cooperative transmission takes place in the duration of  $2\alpha(m - 2)$  symbols. The remaining duration of  $(1 - 2\alpha)(m - 2)$  symbols is allocated to unaided direct transmission from the source to the destination



# Achievable Rates

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- we prove that achievable rate can be obtained by considering the estimate errors as additional sources of Gaussian noise. Since Gaussian noise is the worst uncorrelated additive noise for a Gaussian model achievable rates given in this section can be regarded as worst-case rates
- We obtain the close form of the Achievable Rate for all above five transmission protocols, Due to the lack of space, we do not list them here, In following slides I will give some numerical result based on these achievable rate.

# Training power allocation

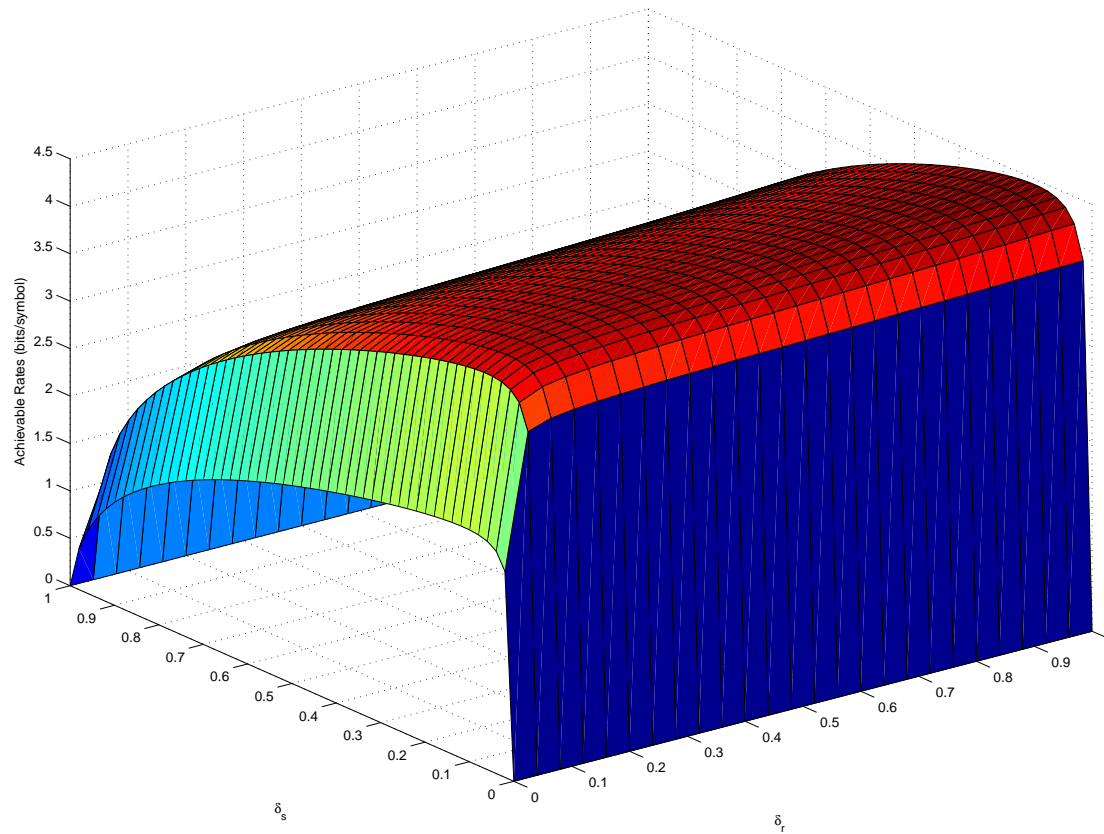


Figure 1: Overlapped AF achievable rates vs.  $\delta_s$  and  $\delta_r$  when  $P_s = P_r = 50$

# Training power allocation

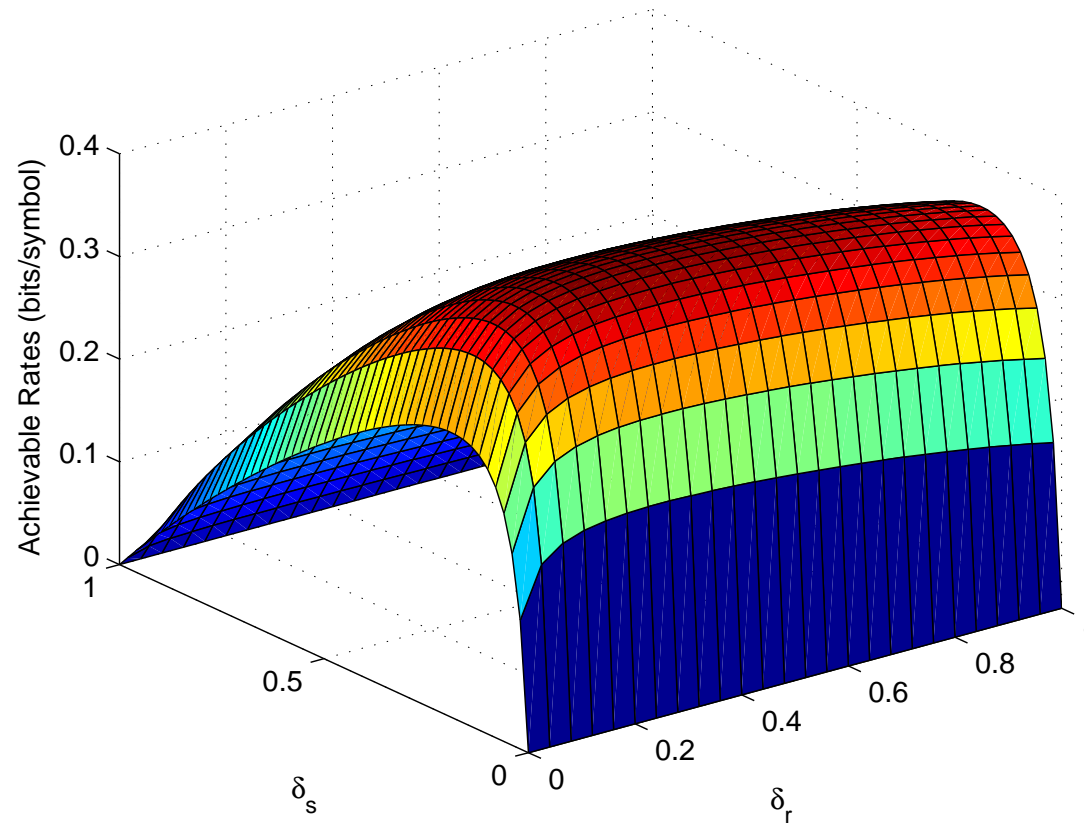


Figure 2: Overlapped AF achievable rates vs.  $\delta_s$  and  $\delta_r$  when  $P_s = P_r = 0.5$

# Degree of cooperation on the performance in AF

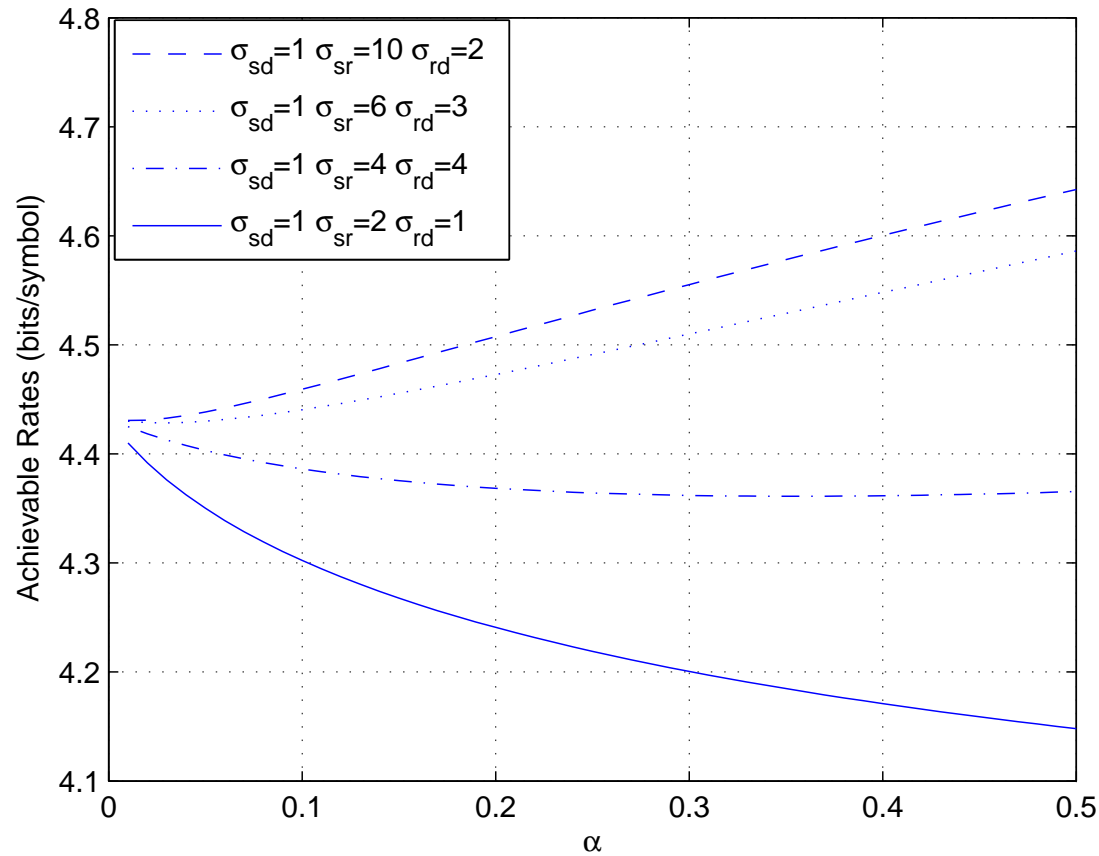


Figure 3: Overlapped AF achievable rate vs.  $\alpha$  when  $P_s = P_r = 50$ ,  $\delta_s = \delta_r = 0.1$ ,  $m = 50$ .

# Degree of cooperation on the performance in AF

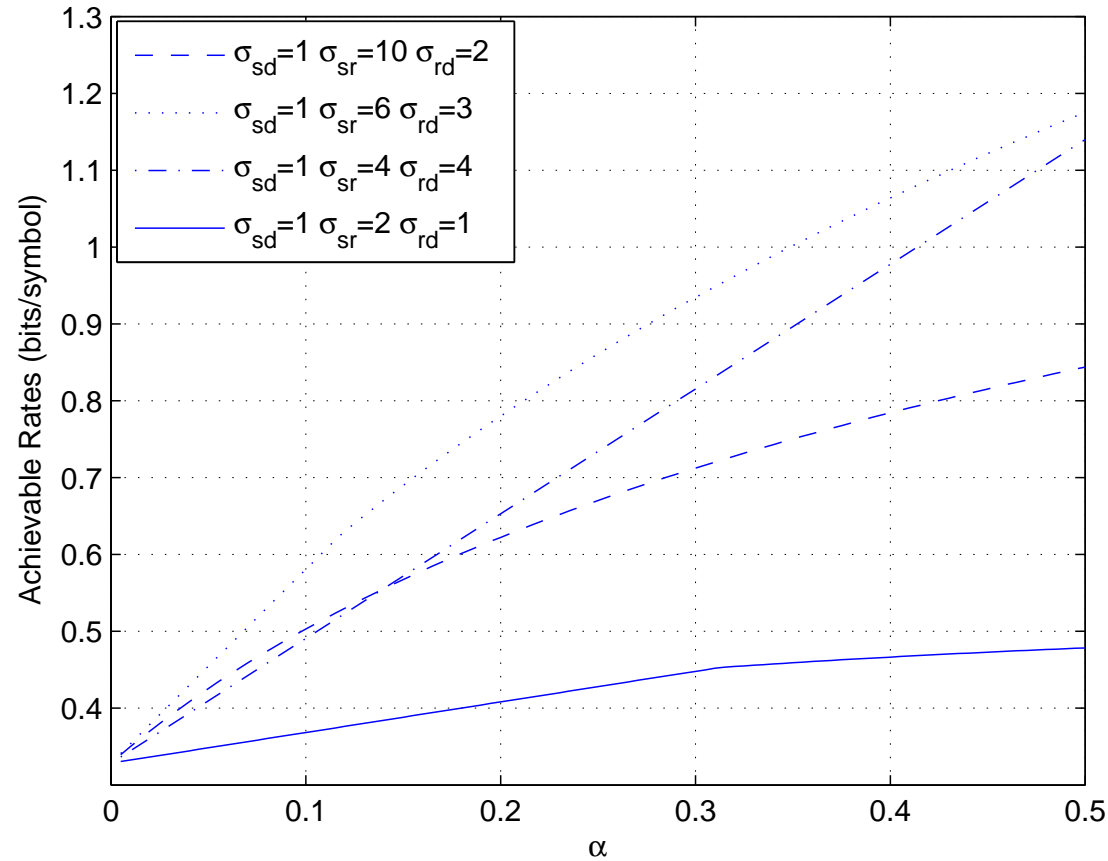


Figure 4: Overlapped DF with repetition coding achievable rate vs.  $\alpha$  when  $P_s = P_r = 0.5, \delta_s = \delta_r = 0.1, m = 50$ .

# Achievable rates of DF parallel channel coding

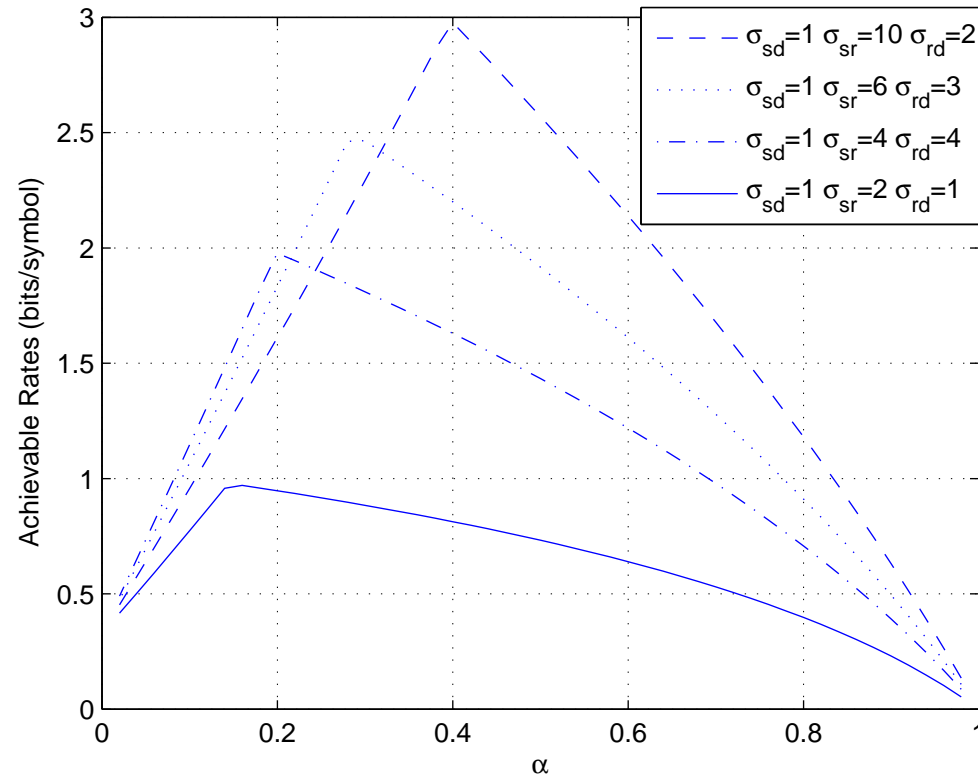


Figure 5: Non-overlapped DF parallel coding achievable rate vs.  $\alpha$  when  $P_s = P_r = 0.5$ ,  $\delta_s = \delta_r = 0.1$ ,  $m = 50$ .

# Power allocation between the source and relay

In certain cases, source and relay are subject to a total power constraint. Here, we introduce the power allocation coefficient  $\theta$ ,  $P_s = \theta P$ ,  $P_r = (1 - \theta)P$ ,

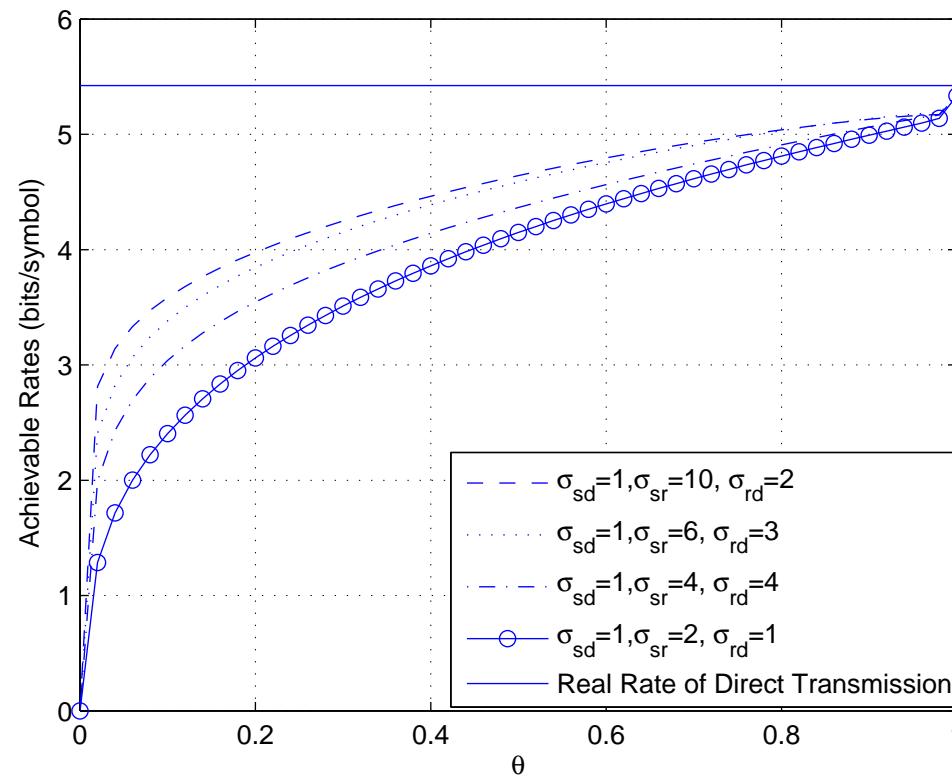


Figure 6: Overlapped AF achievable rate vs.  $\theta$ .  $P = 100$ ,  $m = 50$ .

# Power allocation between the source and relay

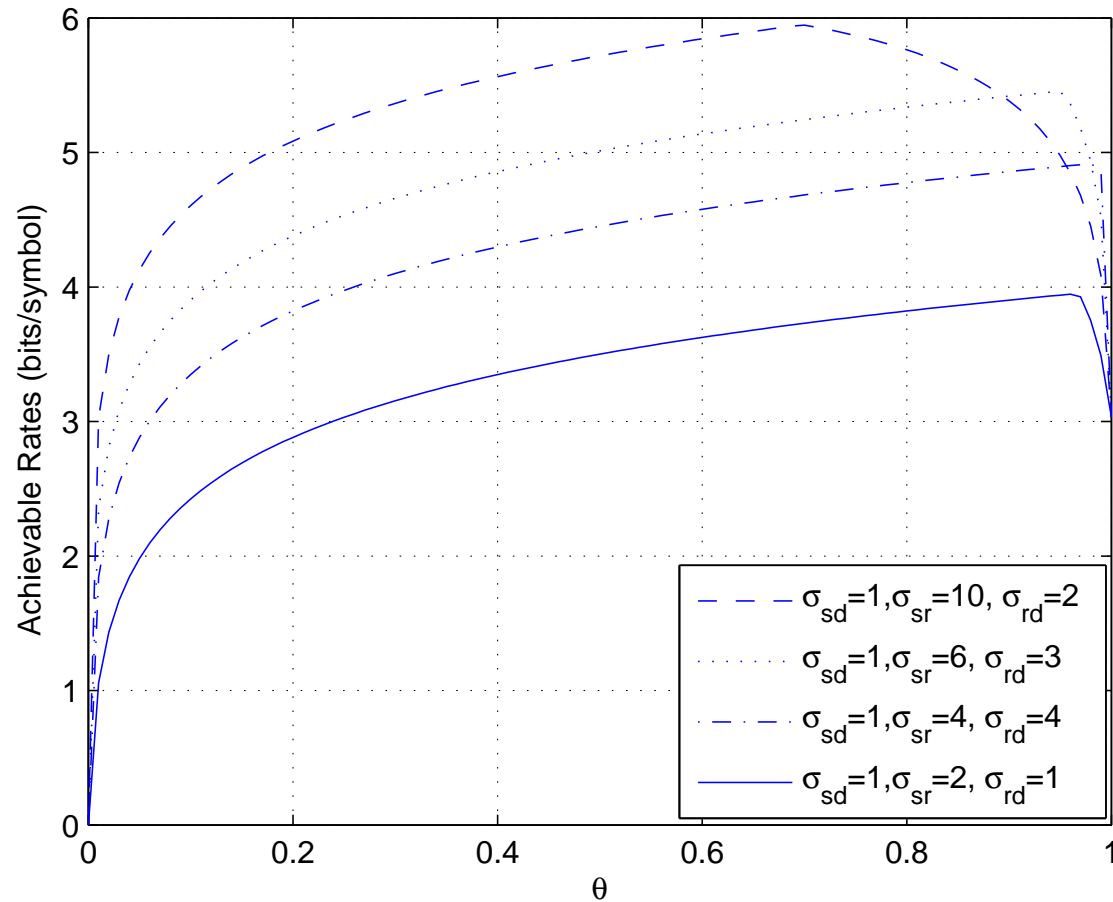


Figure 7: Non-overlapped Parallel coding DF rate vs.  $\theta$ .  $P = 100$ ,  $m = 50$ .



# Conclusion

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- Transmission starts with the training phase. Hence, in our setting, relaying increases the channel uncertainty in the system, and there is increased estimation cost associated with cooperation.
- We have controlled the degree of cooperation by varying the parameter  $\alpha$ . We have observed that if the source-relay channel quality is low, then cooperation is not beneficial and direct transmission should be preferred at high SNRs. On the other hand, we have seen that relaying generally improves the performance at low SNRs. We have noted that DF with parallel coding provides the highest rates.
- Under total power constraints, we have identified the optimal power allocation between the source and relay. We have again pointed out that relaying degrades the performance at high SNRs unless DF with parallel channel