Coding and Decoding for the Dynamic Decode and Forward Relay Protocol

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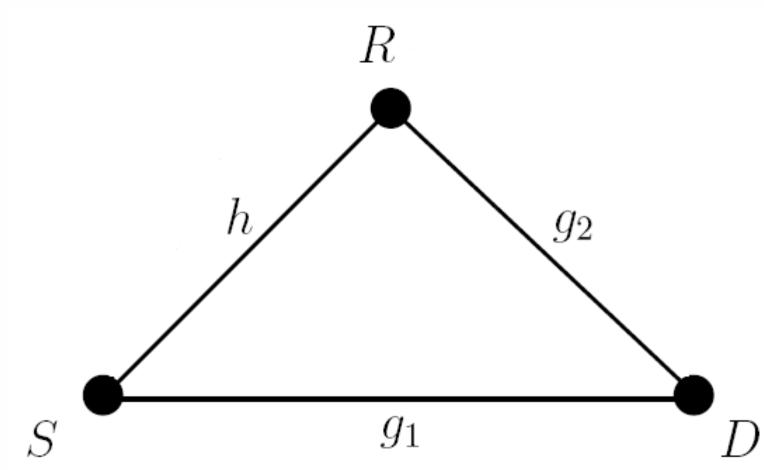
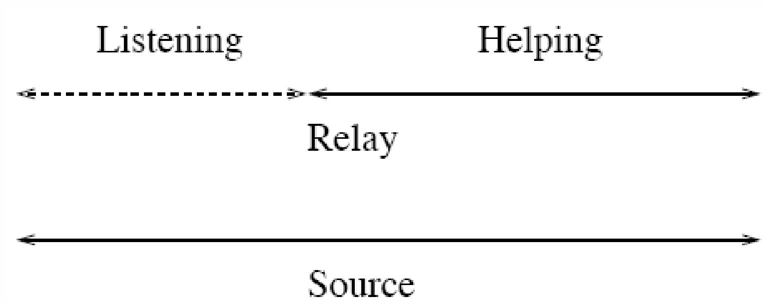
03 June 2008

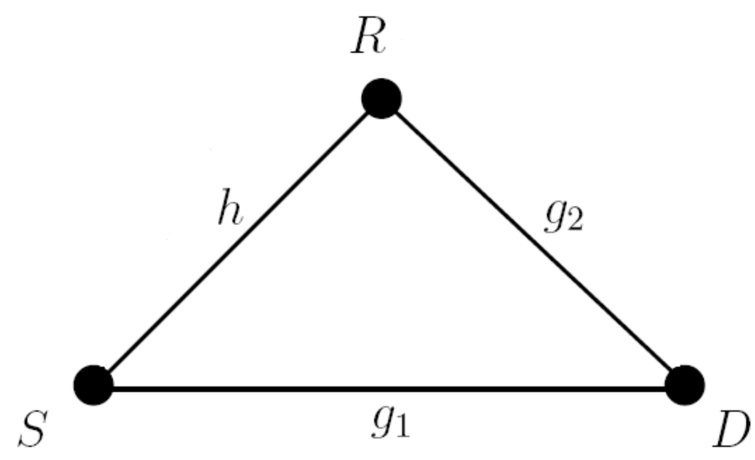
The Relay Channel - Dynamic Decode and Forward



- h, g_1, g_2 are i.i.d. $\mathcal{CN}(0,1)$, constant over a codeword
- Codeword spans M slots of length T symbols each (total block length of MT)
- Power constraint ρ
- Half-duplex operation:



- DDF: Relay decodes after slot number $\mathcal{M} \in \{1, 2, \ldots, M\}$, where $\mathcal{M} = M$ corresponds to "no help" [Azarian et al, 2005]



- Alamouti signalling at the relay [Murugan et al]: through linear processing, D produces the observation

$$\tilde{y}_k = \begin{cases} g_1 x_{s,k} + w_k, & k = 1, \ldots, \mathcal{M}T \\ \sqrt{|g_1|^2 + |g_2|^2} x_{s,k} + \tilde{w}_k, & k = \mathcal{M}T + 1, \ldots, MT \end{cases}$$

Random switch (parallel) channel!

DMT of the DDF Protocol

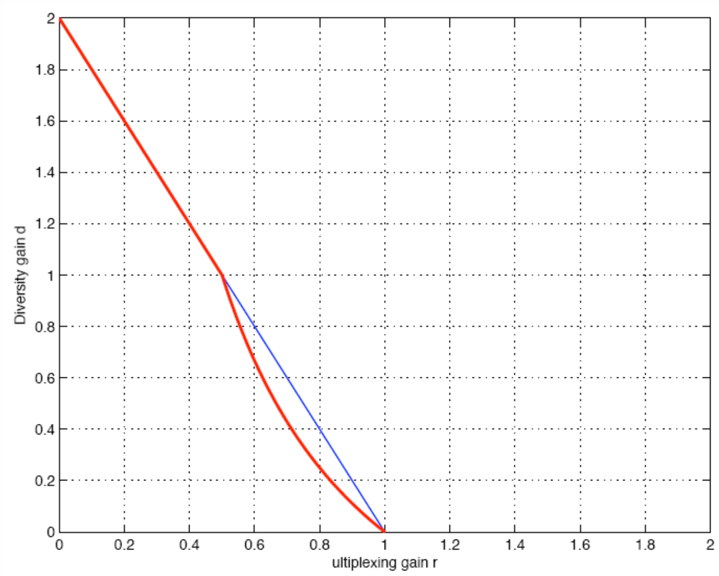
- Metric of performance: Diversity-multiplexing tradeoff (DMT) $d^*(r)$

$$R = r \log \rho$$

$$P_e(r) \doteq \rho^{-d^*(r)}$$

- The DMT of the DDF protocol was analyzed under the assumption $M \to \infty$ and $T \to \infty$ and is given by

$$d^*(r) = \begin{cases} 2(1-r), & 0 \leq r \leq \frac{1}{2} \\ (1-r)/r, & \frac{1}{2} \leq r \leq 1 \end{cases}.$$



What is the DMT for *finite* M and T?

Theorem 1 *The DMT of the single relay DDF scheme with decision times* $m = 1, 2, \ldots, M$ *and finite slot length* $T \geq 1$ *is given by*

$$d_M^*(r) = \min_{1 \leq m \leq M} \left\{ \overline{d}_m(r) + d_m(r) \right\},$$

where

$$\overline{d}_m(r) = \begin{cases} 1 - \frac{Mr}{m-1}, & 0 \leq r \leq \frac{m-1}{M} \\ 0, & \frac{m-1}{M} < r \leq \frac{m}{M} \\ \infty, & \frac{m}{M} < r \leq 1 \end{cases},$$

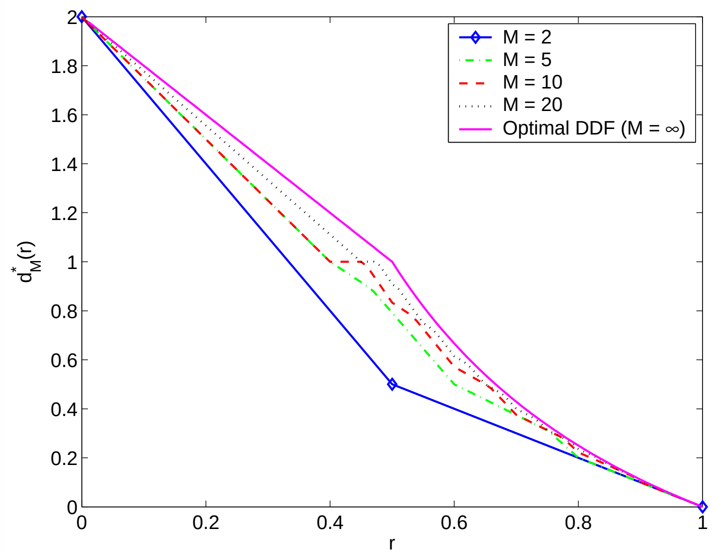
and

$$d_m(r) = \begin{cases} 2 - 2r, & m < \frac{M}{2} \\ \frac{M(1-r)}{m}, & m \geq \frac{M}{2} \end{cases},$$

for $r \geq \frac{1}{2}$

$$d_m(r) = \begin{cases} 2 - 2r, & m < \frac{M}{2} \\ 2 - \frac{rM}{M-m}, & \frac{M}{2} \leq m < M(1-r) \\ \frac{M(1-r)}{m}, & m \geq M(1-r) \end{cases}.$$

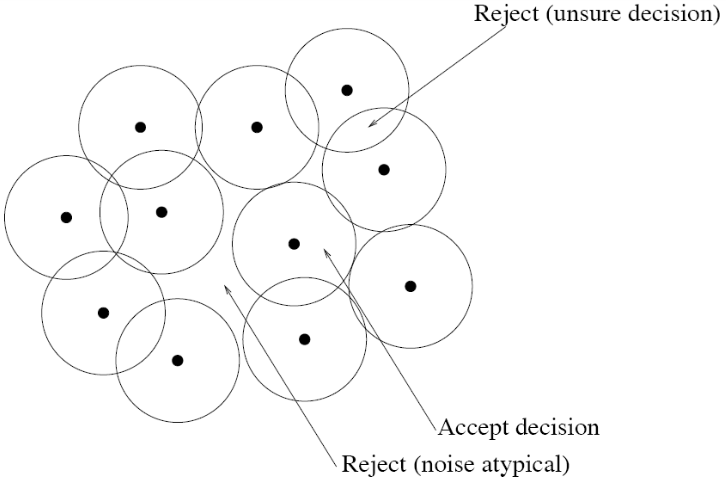
for $r < \frac{1}{2}$.



Outline of the Proof

Proof: (Sketch)

- Upper bound on the DMT:
  - ◇ Assume $T \to \infty$ and that the destination knows $\mathcal{M}$
  - ◇ Compute outage probability of the random switch (parallel) channel
- Achievability for *finite* T, destination does not know $\mathcal{M}$ (i.e., when the relay decodes):
  - ◇ Generate source and relay codes i.i.d. Gaussian
  - ◇ Relay attempts to decode when $S - R$ link is not in outage
    - ∗ No guarantee that this decision is reliable (finite block-length)
    - ∗ Use bounded distance decoding to validate relay's decision



    - ∗ If the bounded distance decoder fails, relay waits for another slot

Outline of the Proof (Contd.)

- Destination decoding: we use of an augmented decoder that simultaneously detects the decision time $\mathcal{M}$ and the information message ω according to the GLRT rule:

$$(\hat{w}, \hat{m}) = \arg\max_{\omega,m} p(\mathbf{y}_0^M | \omega, m, g_1, g_2)$$

- Turns out:
  - ◇ GLRT is optimal, separated detection is sub-optimal!

□

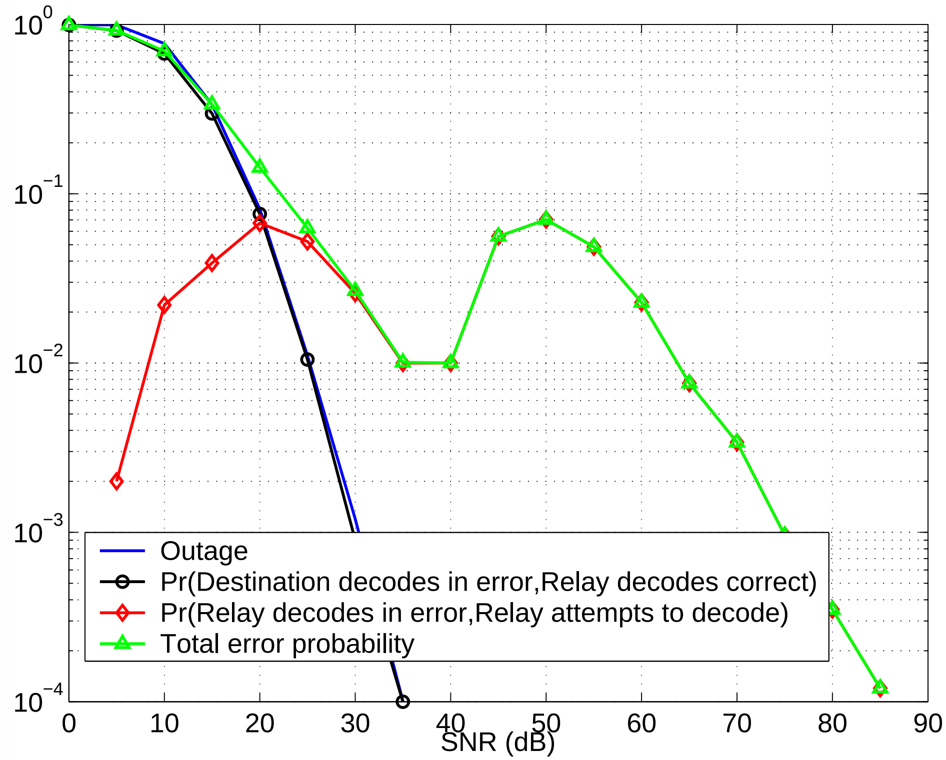
Explicit Construction of DMT Optimal Codes

- Code constructions for the DDF protocol were proposed by [Murugan, Azarian, El Gamal] and [Elia, Kumar]
- We propose: use approximately universal codes for the parallel channel
  - ◇ Algebraic rotations of QAM constellations
  - ◇ Permutation codes
- DMT optimal
- Lower decoding complexity than previous constructions
- Minimum delay construction
- Practical error-detection schemes at the relay

"Standard" decoding rule

- Decode as soon as $S - R$ mutual information exceeds the rate, i.e.,

$$\frac{m}{M}\log(1 + |h|^2\rho) > R$$



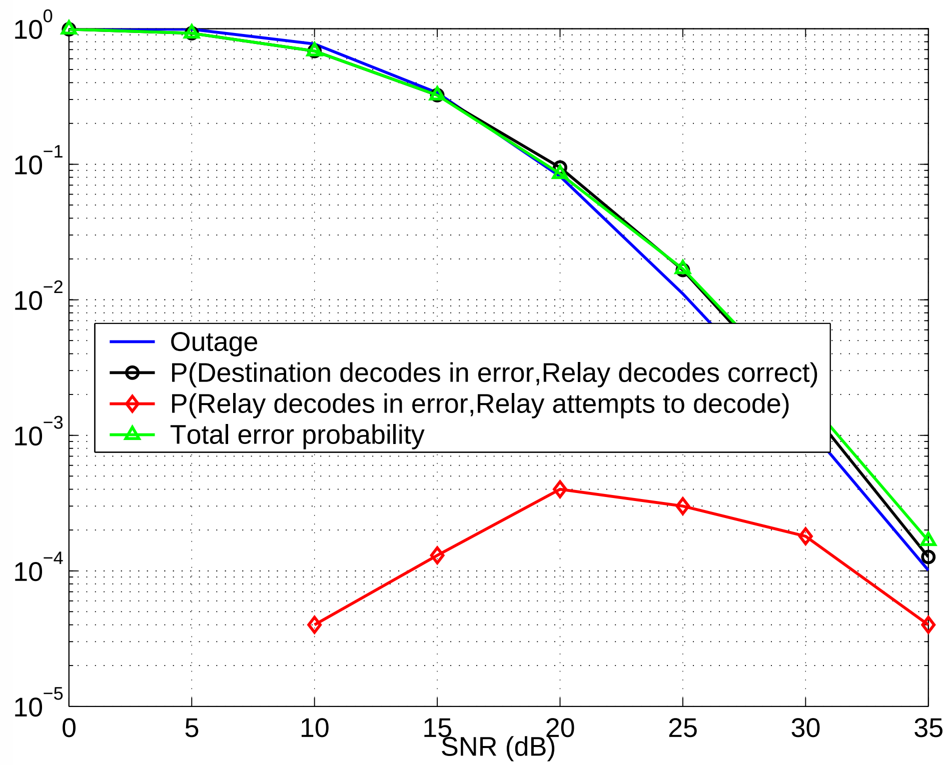
- $T = 1$, $M = 4$, $R = 4$ bpcu

Forney's decoding rule

- Decode tentatively once the $S-R$ MI exceeds the rate
- Accept decision only if "sufficiently reliable", i.e.,

$$\frac{p(\mathbf{y}_{r,0}^m|\widehat{\omega}, h)}{\sum_{\omega\neq\widehat{\omega}} p(\mathbf{y}_{r,0}^m|\omega, h)} \geq \tau,$$

  otherwise wait for the next block
- Proxy for the bounded distance decoder used in the achievability proof



Conclusions

- Characterized the DMT of the DDF protocol for finite block-lengths
  - ◇ No side information necessary to inform the destination of the relay decision time
  - ◇ No CRC needed for error detection at the relay
- Constructed minimum-delay DMT optimal codes that have excellent error probability performance
- More details:
  - ◇ to be presented in ISIT-2008, Toronto!
  - ◇ See arxiv submission