# Capacity of a Class of Diamond Channels

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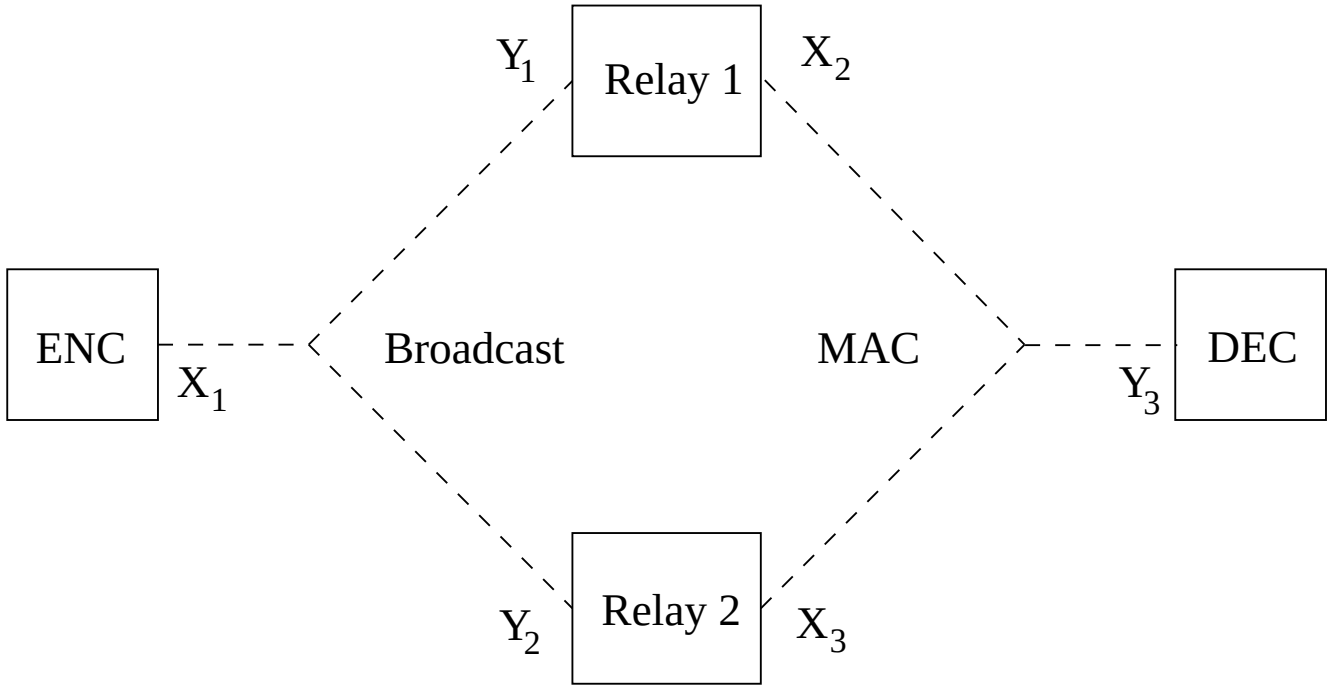
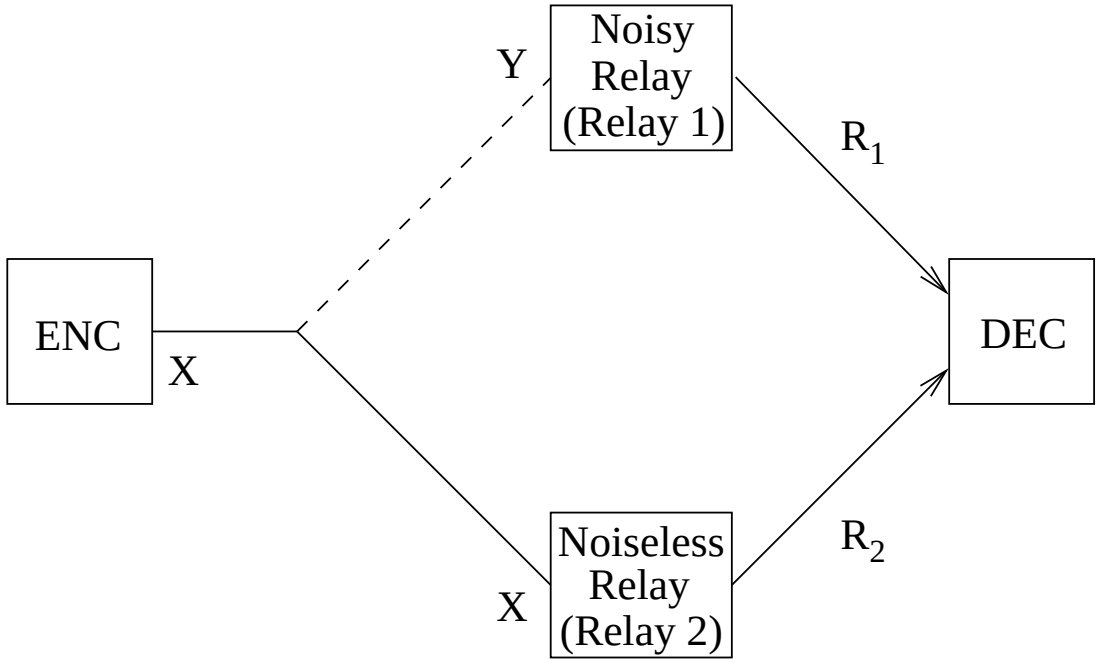
# Diamond Channel

- Diamond channel was first proposed and studied by Schein in 2001.
- Diamond channel consists of a transmitter, a destination and two relays.
- Cascade of a broadcast channel and a MAC channel.
- There is no direct link between the transmitter and the destination.



# A Special Class of Diamond Channels

- Schein studied a special class of diamond channels.
  - MAC replaced by two orthogonal links with rates $R_1$ and $R_2$.
  - One branch of the broadcast channel is noiseless, i.e., $Y_2 = X$.
- Schein proposed two achievable schemes
  - Partial decoding at the noisy relay.
  - Compression at the noisy relay without decoding.



## Main Result

The rate triple $(R, R_1, R_2)$ is achievable in the above channel if and only if the following conditions are satisfied

$$\begin{aligned} R &\leq I(U;Y) + H(X|U) \\ R_1 &\geq I(Z;Y|U,X) \\ R_2 &\geq H(X|Z,U) \\ R_1 + R_2 &\geq R + I(Y;Z|X,U) \end{aligned}$$

for some joint distribution

$$p(u,z,x,y) = p(u,x)p(y|x)p(z|u,y)$$

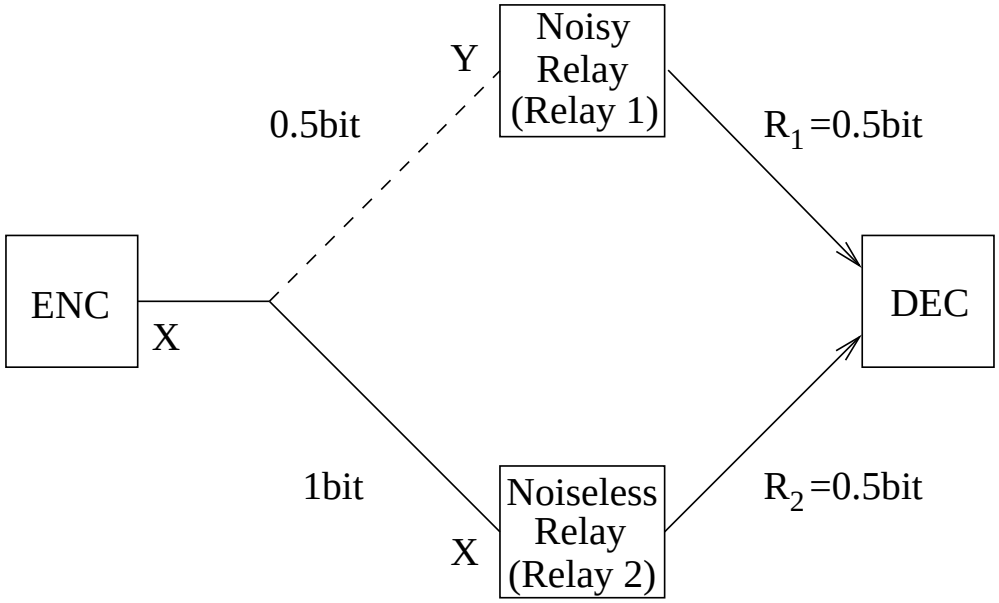
**Remark**: Superposition code; Noisy relay decodes the inner code and compresses the outer code.

## Comparing with the Cut-set Bound

- Capacity is strictly less than the cut-set bound

$$R_1 + R_2 \geq R + I(Y; Z|X, U) \Longleftrightarrow R \leq R_1 + R_2 - I(Y; Z|X, U)$$

- A BSC example. $Y = X + Z$, $H(Z) = 0.5$ bit and $R_1 = R_2 = 0.5$ bit.
- Cut-set bound yields 1 bit.
- Noisy relay cannot decode 0.5 bit with $R = 1$ bit.



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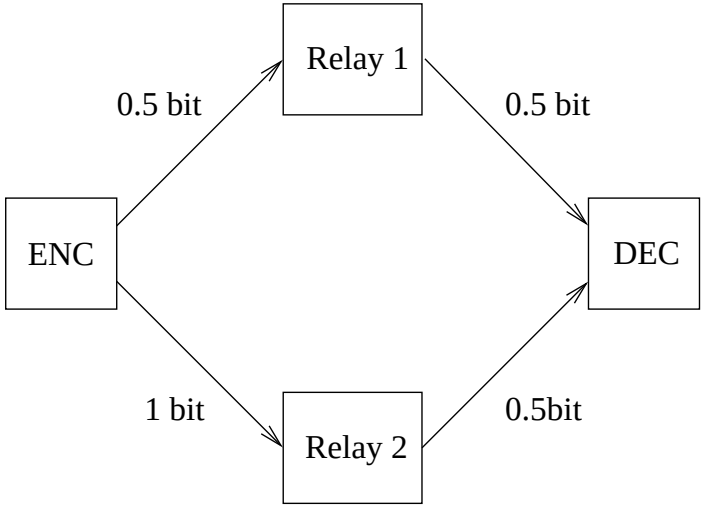
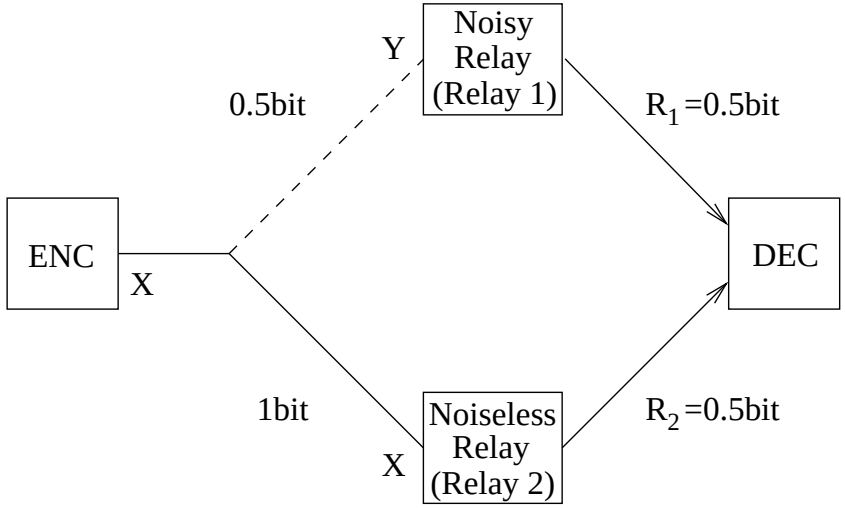
- A BSC example. $Y = X + Z$, $H(Z) = 0.5$ bit and $R_1 = R_2 = 0.5$ bit.
- Cut-set bound yields 1 bit, which is not achievable.
- Introducing a broadcast node into a wired network may reduce the capacity.



## A Relay Perpective

- Decode-And-Forward (DAF) is optimal:
  - Cover-El Gamal: Degraded relay channel.
- Partial DAF is optimal:
  - El Gamal-Aref: Semi-deterministic relay channel.
  - El Gamal-Zahedi: Relay channel with orthogonal transmitter-relay link.
- Compress-And-Forward (CAF) is optimal:
  - Kim: A special relay channel with orthogonal relay-destination link.
  - Aleksic-Razaghi-Yu: A special Ahlswede-Han channel.
- We are the first to show the optimality of the combination of DAF and CAF in a certain channel, which though is not a relay channel in the strict sense.