Partial iterative signal processing at iterative receivers

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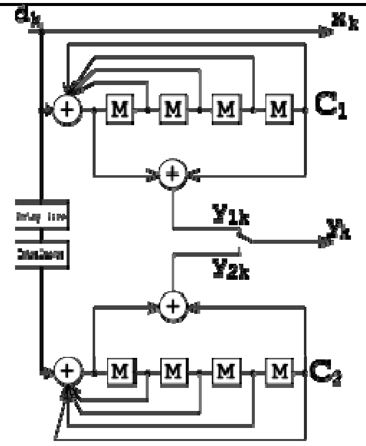
Background

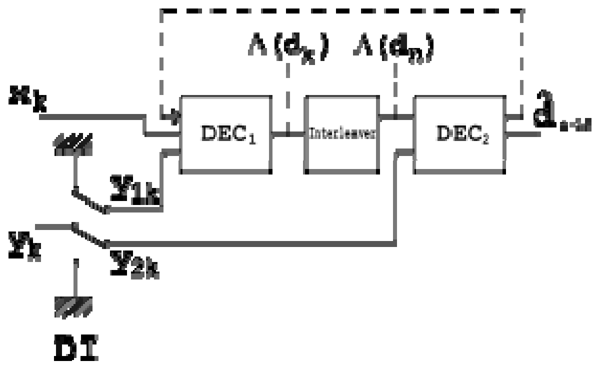
- Iterative receiver designs has been a key technique to provide near-limit performance for advanced wireless communication systems

- A major disadvantage in applying these advanced techniques: PROHIBITIVE PROCESSING COMPLEXITY

- Example: Iterative decoding for turbo codes

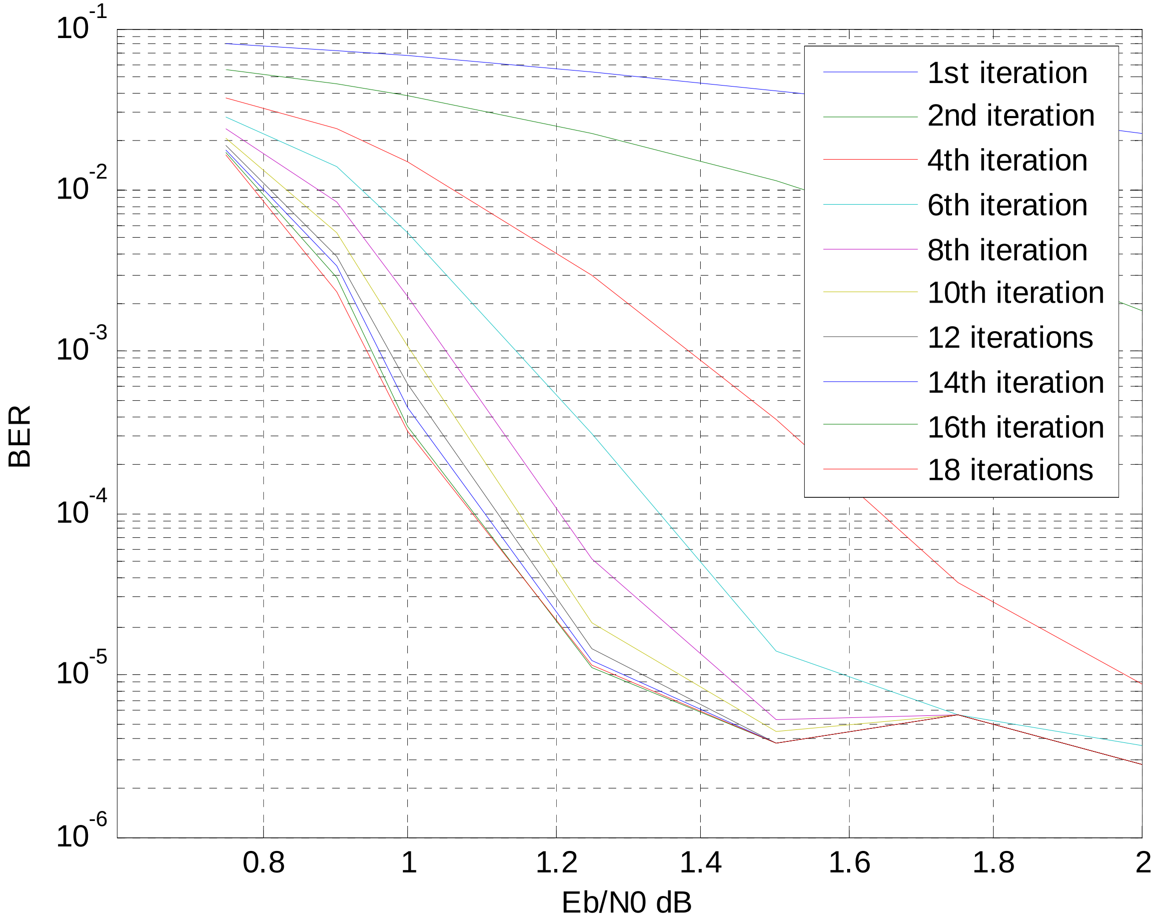


A turbo encoder



A turbo decoder

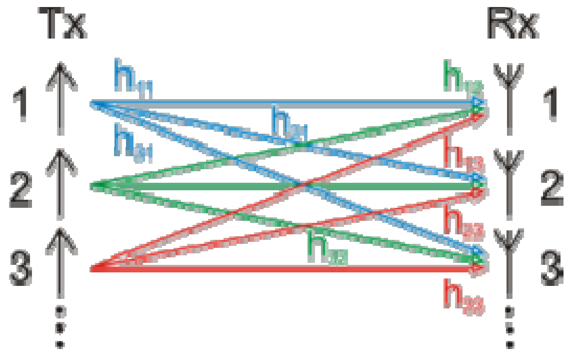
Decoding performance: Diminishing iterative gain as iteration number increases



Log-MAP decoding example: punctured, rate ½, (7,5) RSC encoders, random interleaver size 8192.

More general scenarios in wireless communications

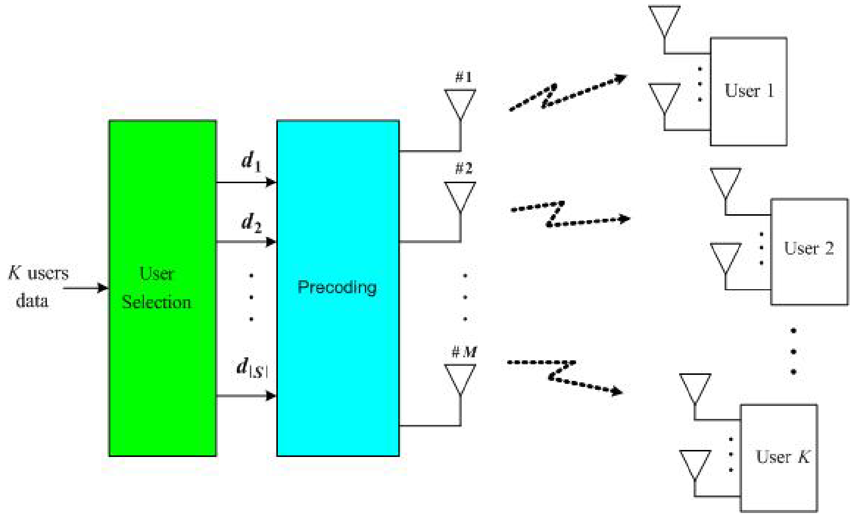
- Multi-input multi-output (MIMO) systems



- Multi-user communication systems

- … …

- Using conventional iterative signal processing schemes to achieve near optimal performance, computational complexity increases exponentially to both the number of user antennas and the number of users.



Existing complexity saving methods -- Iteration stop rules: stop iterations when little decoding gain is likely from more iterations.

- **Cross-entropy measurement:** measures the relative information between the binary data frame's a posteriori probability distributions produced by component decoders. ([5] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *IEEE Trans. Inform. Theory*, vol. 42, no. 2, pp. 429–445, Feb. 1996.)
- **Risk function:** estimates the frame error probability via extrinsic ***L***-values. ([2] G. Bauch, H. Khorram, and J. Hagenauer, .Iterative equalization and decoding in mobile communications systems,.
in *Proc. European Personal Mobile Commun. Conf.*, Bonn, Germany, Sept./Oct. 1997.)
- **Hard decision comparison:** check hard decision difference after each iteration. ([6] S. Lin and D. J. Costello Jr.., *Error Control Coding*, Englewood Cliffs, NJ, Prentice-Hall, 2nd edition, 2004.)

Early detection and trellis splicing: delete symbols with reliable hard decisions and reconnect the left symbols to form a shortened trellis. Measurement by absolute values of extrinsic information.

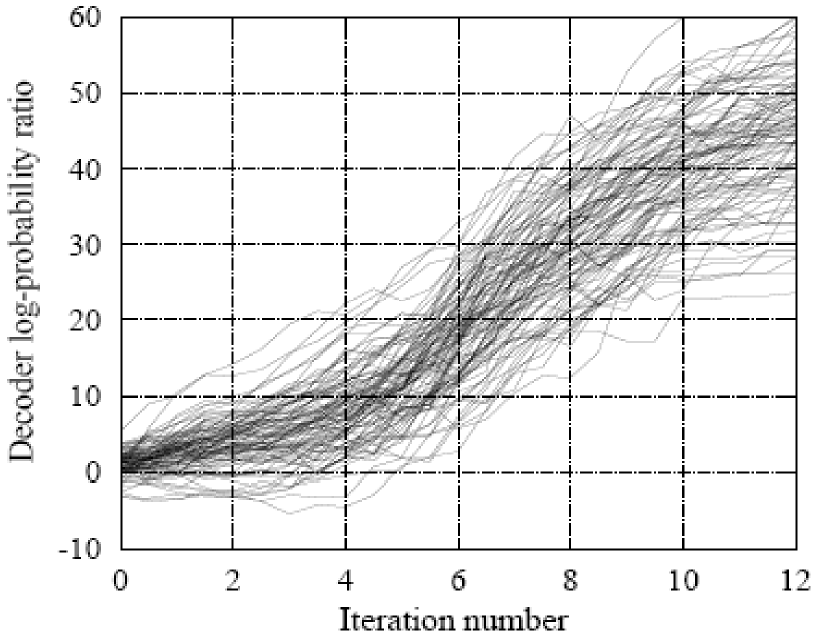


Fig. 1. A plot of the log-probability ratio versus iteration number, for the correct value of each information bit in a randomly selected set of 100 bits *within the same block* of 10,000 bits.

([4] B. J. Frey and F. R. Kschischang, .Early detection and trellis splicing: reduced-complexity iterative decoding,. *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 2, pp. 153.159, Feb. 1998.)

Dilemma in block stopped iterations: a few correctable errors, or several more iterations.

Can we spend just a little more computational power to correct the last errors? – Symbol stop rule.

Considering symbol based processing:

- Question 1: How to select symbols?
- Question 2: How to adjust computations for these selected symbols?

- A novel approach – convergence detection based symbol selection and partial iterative processing – reduce complexity without sacrificing performance !

Detection Convergence Measurement ([5] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *IEEE Trans. Inform. Theory*, vol. 42, no. 2, pp. 429–445, Feb. 1996.)

Definition of relative information between two PDF's:

$$T_{block} = E_P\{\log \frac{P(\mathrm{x})}{Q(\mathrm{x})}\} = \sum_{n=0}^{N-1} E_P\{\log \frac{P(x(n))}{Q(x(n))}\}$$

Relative information between symbol distribution from two component decoders

$$T^i(n) \approx \frac{|\Delta L_a^i(n)|^2}{\exp\left(|L_Q^i(n)|\right)}$$

$$\Delta L_a^i(n) = L_a^i(n) - L_a^{i-1}(n)$$

$$L_Q^i(n) = L_e^i(n) + L_a^{i-1}(n)$$

Assuming i.i.d. among symbol distributions after each iteration (approximately).

$$T_{block}^i \approx \sum_{n=1}^{N} T^i(n)$$

BCJR MAP decoding: ([1] L. R. Bahl, J. Cocke, F. Jelinek, , and J. Raviv, .Optimal decoding of linear codes for minimizing symbol error rate,. *IEEE Trans. Inform. Theory*, vol. 22, pp. 284.287, Mar. 1974.)

– $y_j(n) = a_j(n)c_j(n) + v_j(n)$

$$L(n) = L(x(n) \mid \mathbf{y})$$

$$= \ln \frac{\sum_{(S',S)\Rightarrow x(n)=+1} \alpha_{n-1}(S')\gamma_n(S',S)\beta_n(S)}{\sum_{(S',S)\Rightarrow x(n)=-1} \alpha_{n-1}(S')\gamma_n(S',S)\beta_n(S)}$$

$$L_e(n) = L(n) - L_a(n)$$

$$\alpha_{n-1}(S') = P(S', \mathbf{y}_{j<n})$$

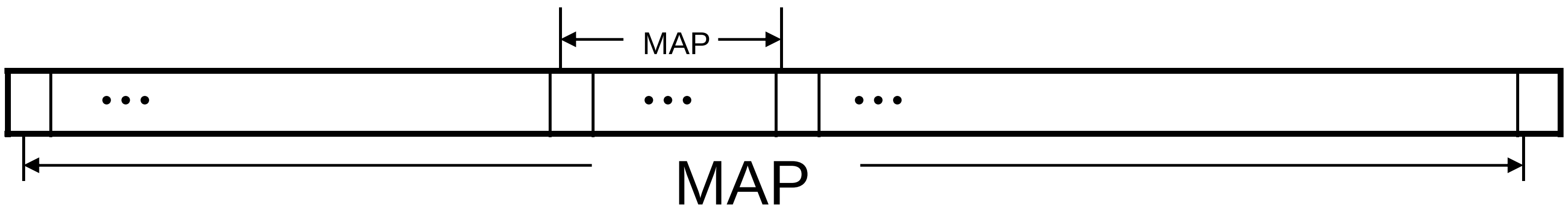
$$\beta_n(S) = P(\mathbf{y}_{j>n} \mid S)$$

$$\gamma_n(S', S) = P(y(n), S \mid S').$$

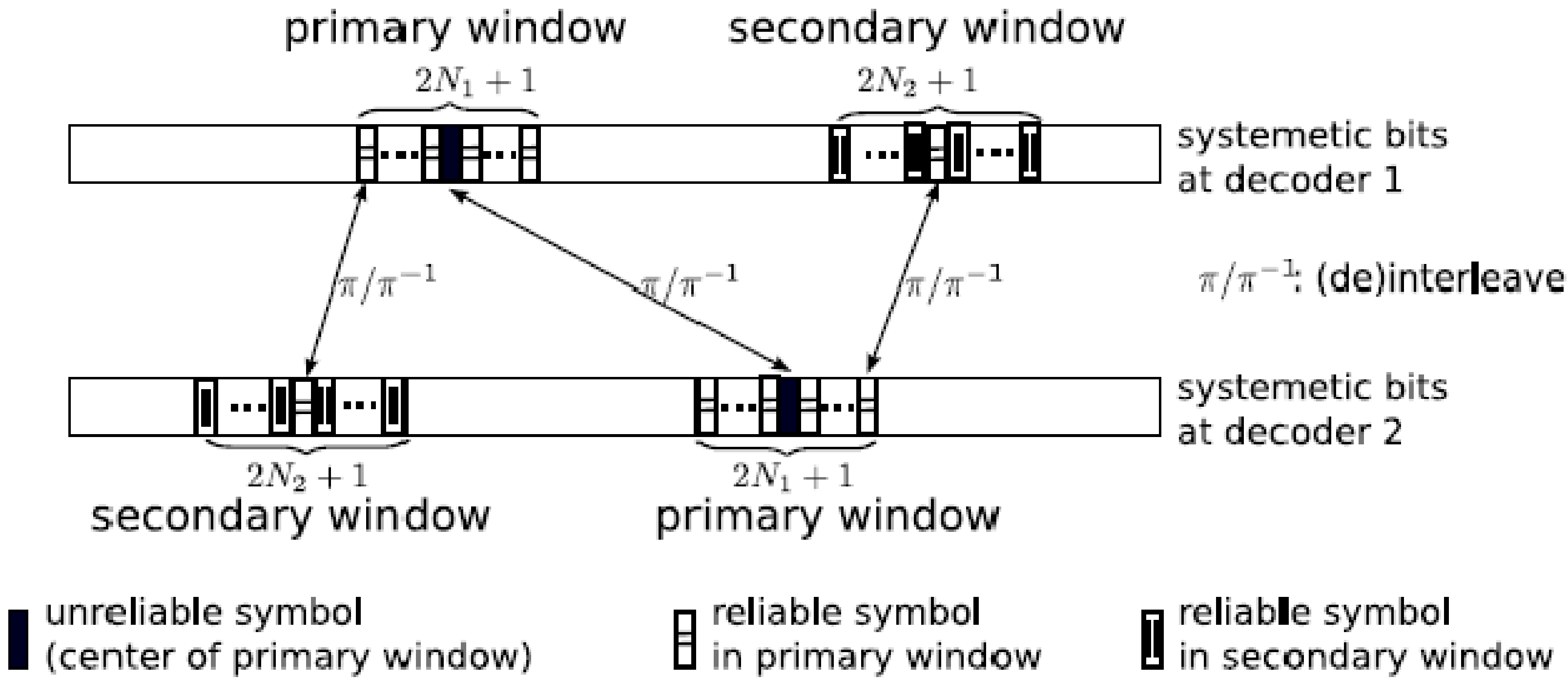
$$\alpha_n(S) = \sum_{S'} \gamma_n(S', S)\alpha_{n-1}(S')$$

$$\beta_{n-1}(S') = \sum_{S} \beta_n(S)\gamma_n(S', S)$$

$$\gamma_n(S', S) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{j=0}^{1}[y_j(n) - c_j(n)]^2\right) P(x(n))$$



Partial iterative MAP decoding



Complexity by a partial iteration: a loose upper bound

$$\frac{C_{\text{partial}}}{C_{\text{complete}}} \leq \frac{M(2N_1(2N_2+1)+2N_1+1)}{N} = \frac{M(4N_1(N_2+1)+1)}{N}$$

Symbol selection and partial iterative decoding process

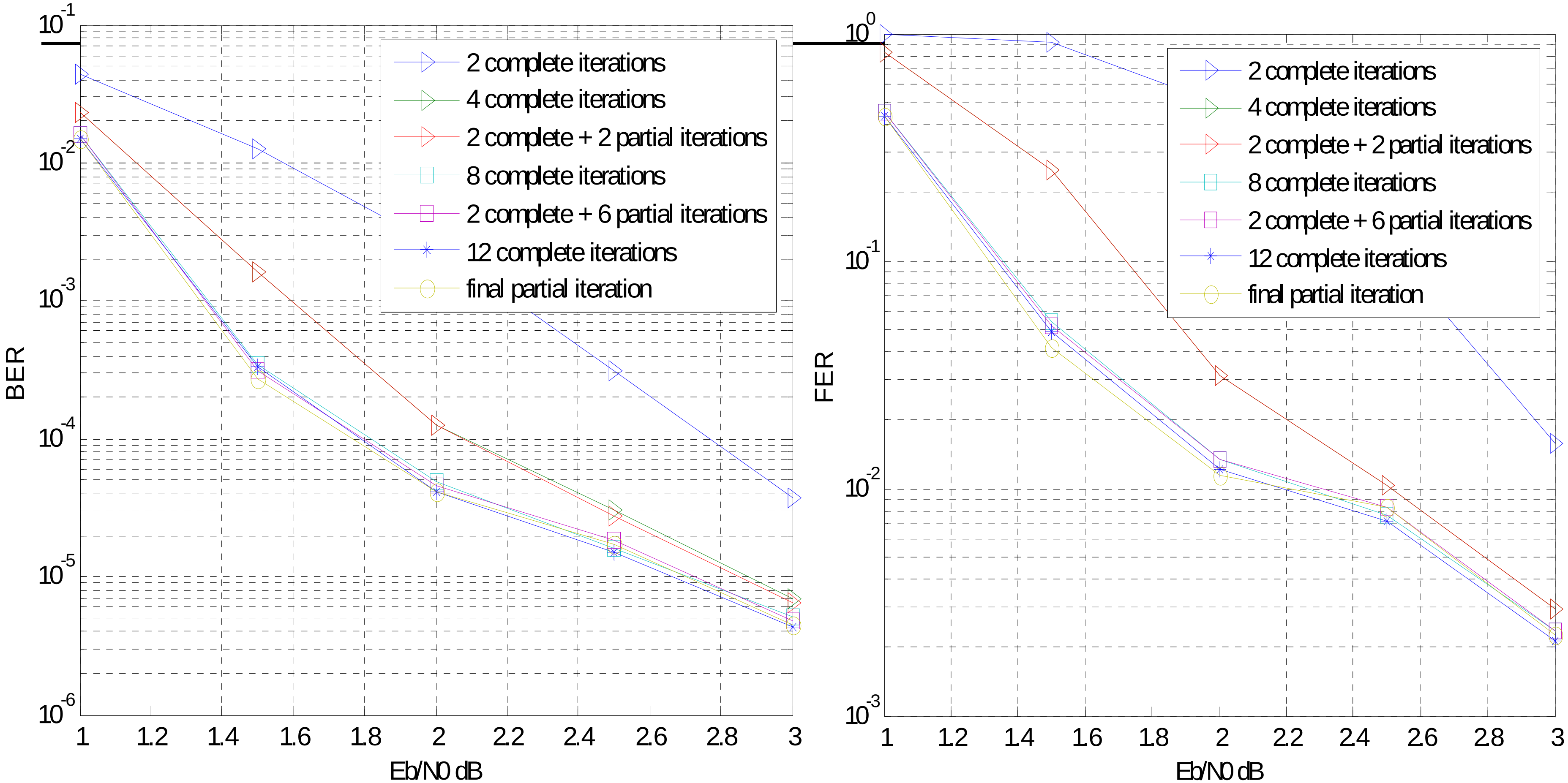
- 1) Select: $S1$ = {unreliable symbols at decoder 1}
  $S2$ = {unreliable symbols at decoder 2}
  If S1 is empty, stop iterations and make hard decisions.

- 2) Apply primary windows:
  $W1$ = {symbols in primary windows at decoder 1}
  $W2$ = {symbols in primary windows at decoder 2}

- 3) Apply secondary windows after interleaving symbols from primary windows
  $U1$ = {symbols in primary or secondary windows at decoder 1}
  $U2$ = {symbols in primary or secondary windows at decoder 2}

- 4) For each decoder i, $i = 1, 2$, and for $n = 1, 2, \ldots, N$, if symbol *n is in Ui*, compute the LLR values according to the symbol MAP formula. Otherwise, *soft* values are kept the same as in previous iteration. Go back to (1).

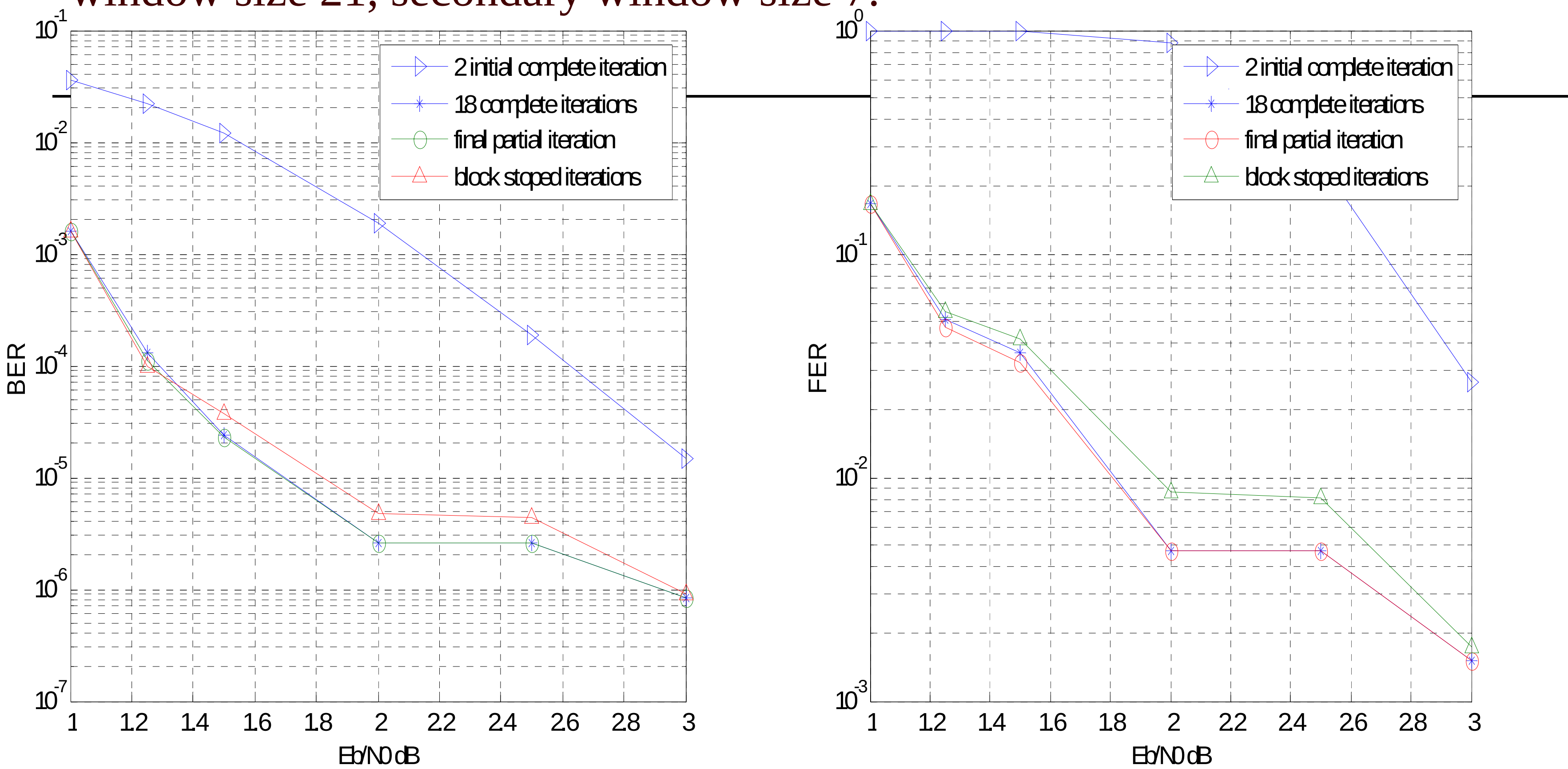
Simulation I: 2 fixed initial complete iterations + partial iterations. AWGN channel, punctured rate ½ (7, 5) RSC encoders, interleaver size 1024. Primary window size 15, secondary window size 3.



Total complexity equivalent to complete iterations:

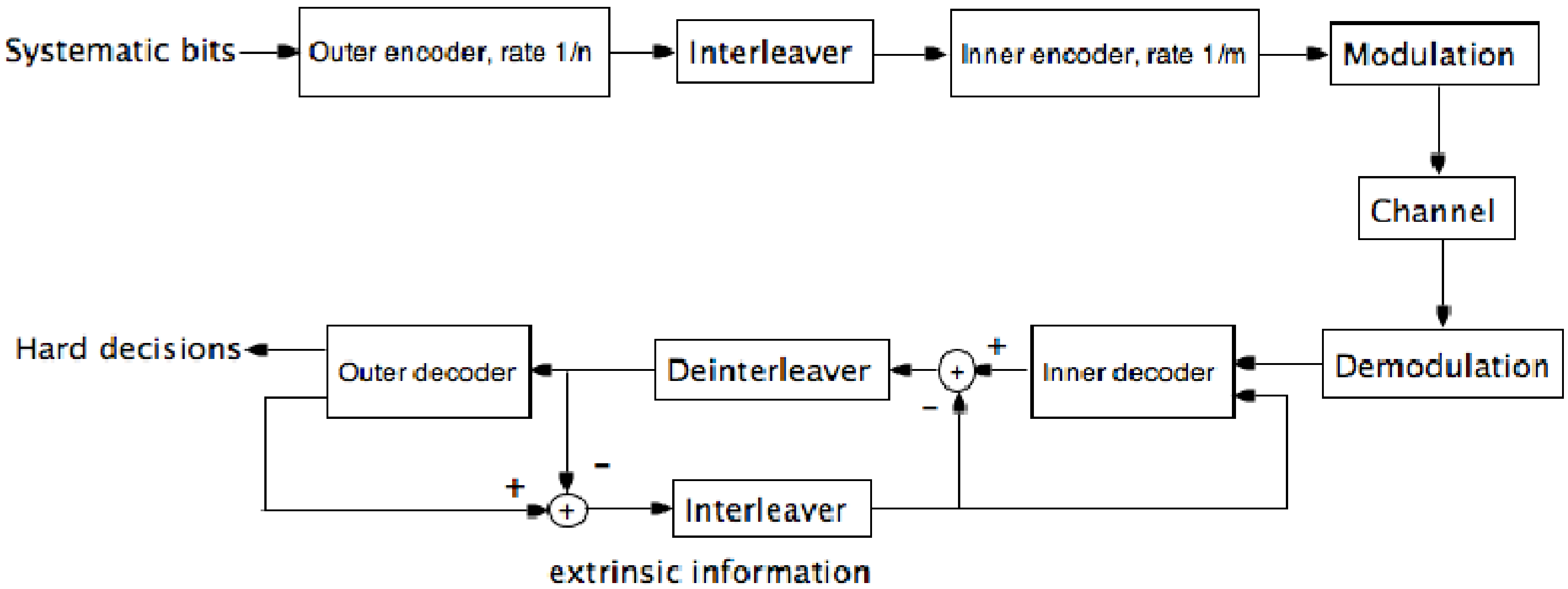
SNR (dB)	1.0	1.5	2.0	2.5	3.0
Complexity	9.57	5.89	4.21	3.33	2.97

Simulation II: Block stopped complete iterations + partial iterations. AWGN channel, punctured rate ½ (7, 5) RSC encoders, interleaver size 4096. Primary window size 21, secondary window size 7.



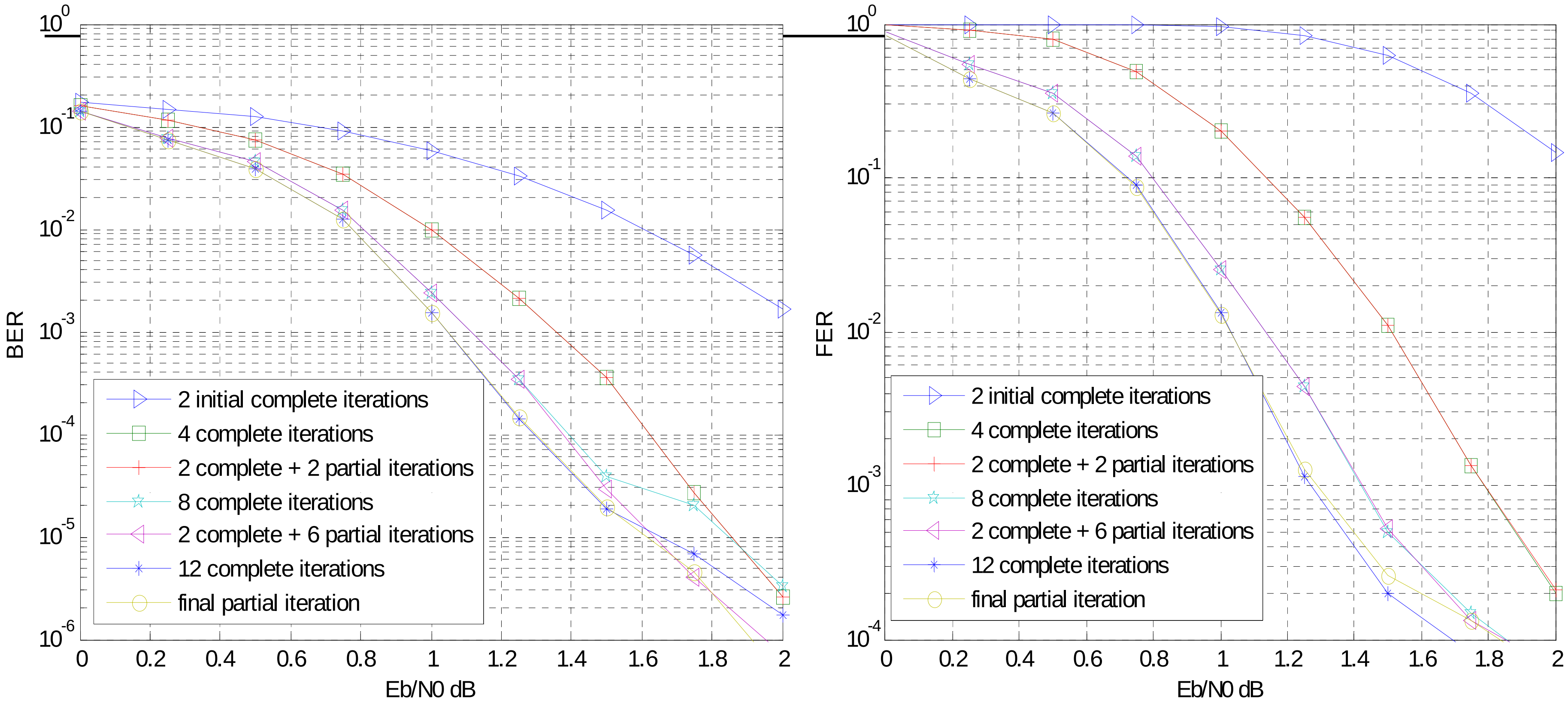
SNR (dB)	1.0	1.25	1.5	2.0	2.5	3.0
Complexity by block stopped iterations	11.00	7.11	5.59	3.99	3.14	2.91
Complexity by block stopped iterations + partial iterations	11.79	7.91	6.17	4.36	3.44	3.05
Extra complexity by partial iterations	0.79	0.80	0.58	0.37	0.30	0.14

Partial iterative processing example: SCCC system



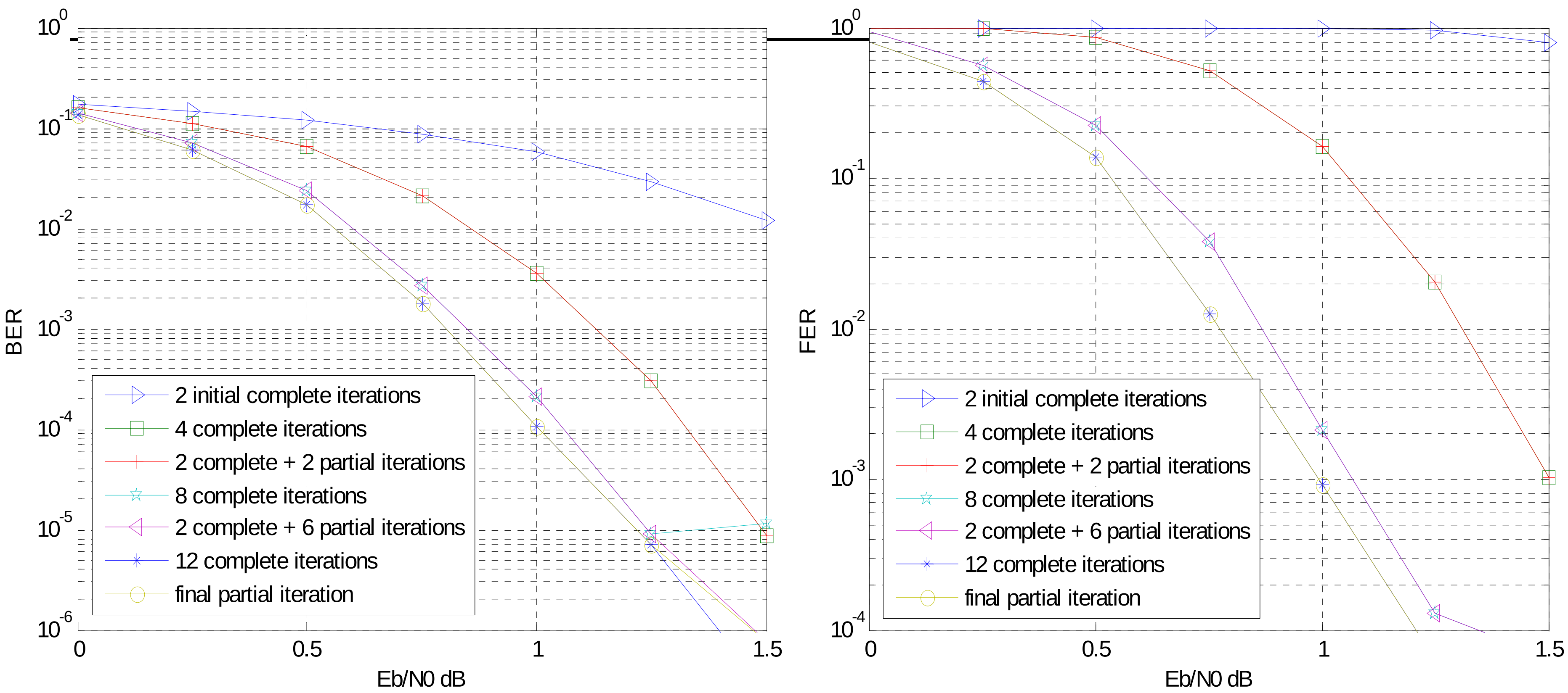
Simulation I: Bit error rates. 2 initial complete iterations + partial iterations.

AWGN channel, unpunctured rate 1/4 (7, 5) RSC encoders, interleaver size 1024.
Primary window size 15, secondary window size 3.



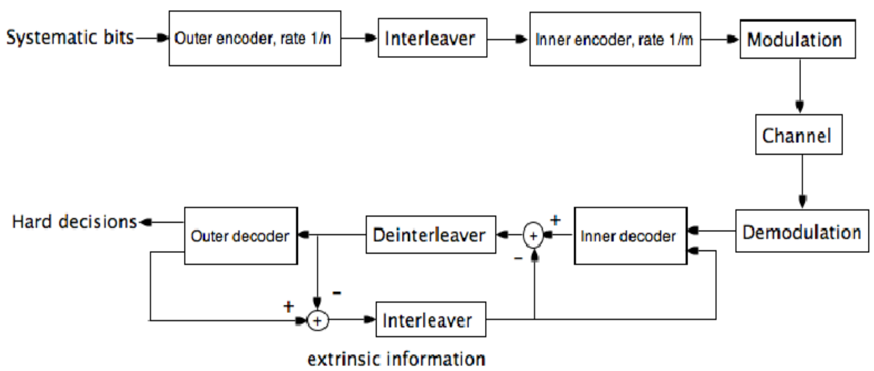
SNR (dB)	0	0.5	1	1.5	2
Complexity by 12 complete iterations	12 C	12 C	12 C	12 C	12 C
Complexity by 2 initial complete iterations + partial iterations	11.58 C	8.62 C	5.36 C	4.11 C	3.40 C

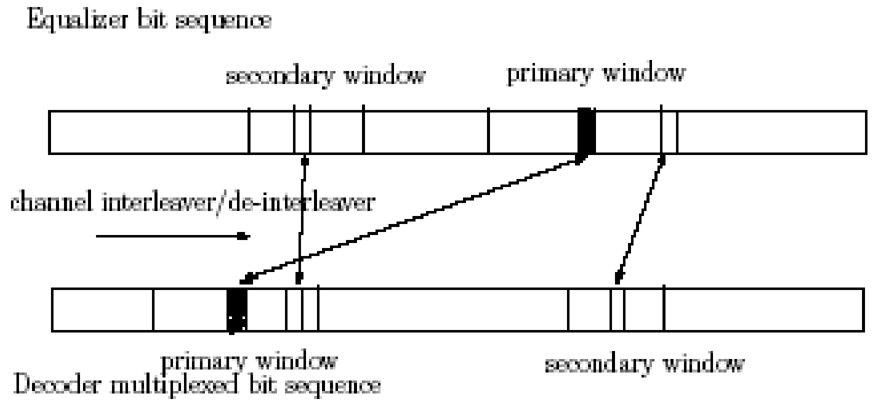
Simulation II: Bit error rates. Block stopped complete iterations + partial iterations. AWGN channel, unpunctured rate ¼ (7, 5) RSC encoders, interleaver size 2048. Primary window size 15, secondary window size 3.



SNR (dB)	0.5	0.75	1	1.25	1.5
Complexity by 12 complete iterations	12 C	12 C	12 C	12 C	12 C
Complexity by 2 initial complete iterations + partial iterations	8.23 C	6.24 C	5.17 C	4.47 C	4.07 C

Partial iterative processing example: Joint partial iterative equalization and channel decoding





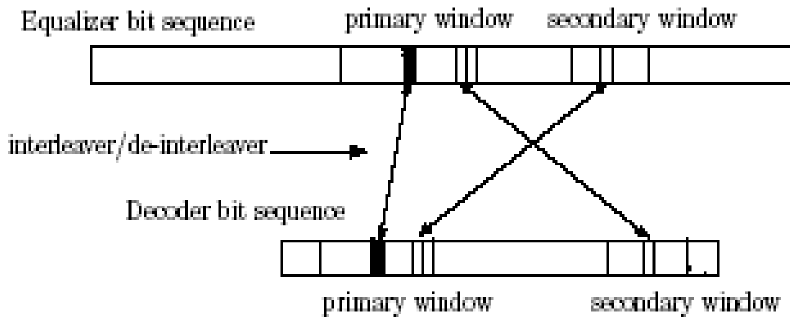


Fig. 1. Bit sequences and windows deployments for receivers with a single decoder and an equalzier.

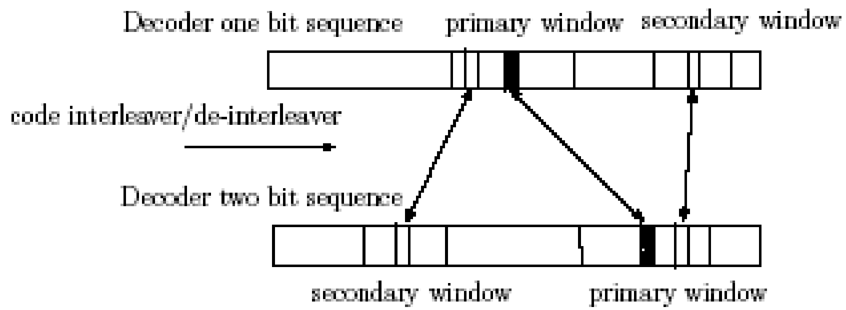


Fig. 2. Bit sequences and windows deployments for receivers with two component decoders and an equalzier.

Partial iterative equalization and decoding performance

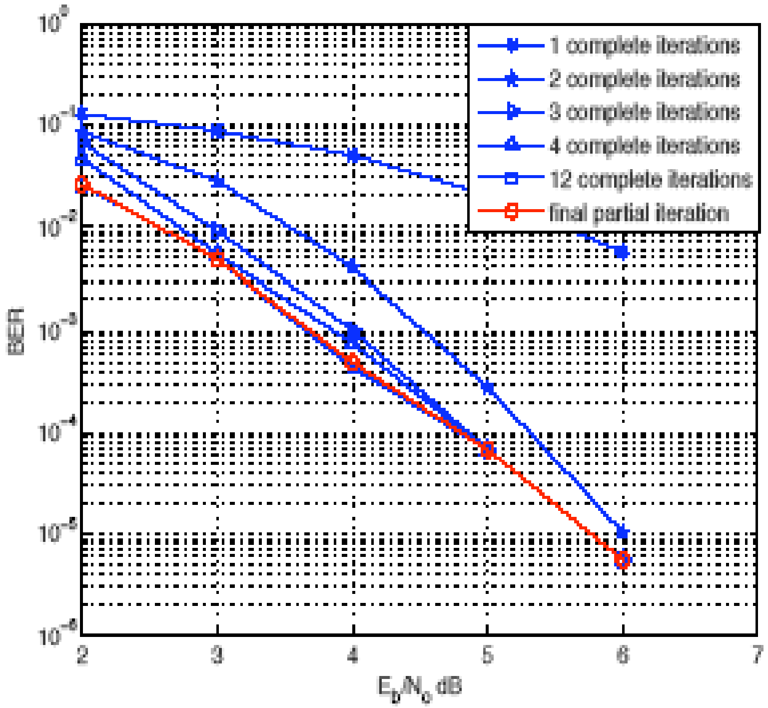


Fig. 3. BER comparison for case I: partial iterations and complete iterations. A single convolutional decoder with an equalzier. Channel interleaver length 2048.

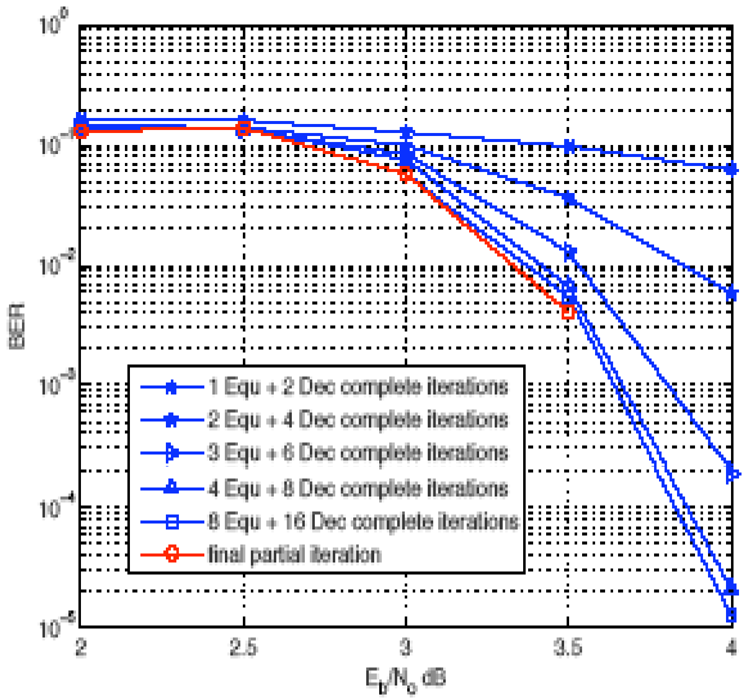


Fig. 4. BER comparison for case II: partial iterations and complete iterations. Two component decoders with an equalzier. Code interleaver length 1024. Channel interleaver length 2050.

Conclusions and future works

- A MAP detection with reduced block length is valid given the convergence of boundary symbols.
- Cross-entropy between the component detector/decoders' a posteriori distributions is effective in symbol selection. For serial concatenated modules, coded bits of the outer encoder can be classified for symbol selection. For turbo coded transmissions, the information bits can be used for classification instead. Applying windows around unreliable symbols, partial iterations well maintain the performance by complete iterations with a small fraction of the original complexity.
- Initial complete iterations are necessary for partial iterations' performance. 2 initial iterations are sufficient for short length frames; for long frames an adaptive number of initial iterations may incorporate block stop rules to ensure the convergence of most symbols.
- The serial concatenation framework can be extended to more general cases to include iterative equalization, MIMO detection, multi-user detection, etc. Benefits of partial iterations for multi-stage iterative receivers to be investigated.