Compressive Recognition System Design and Analysis

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Why Compressive Recognition?

- Bandwidth constraints of network systems where sensing agents, database and the action agents are distributed over different locations.
- Resource constraints in animals and machines:
  - –number of neurons, blood supply, ATP, etc.
  - –cost of hardware.
  - –time constraint for data processing and computation.
- Redundancy of input data.
- There is noise in the environment.

What to consider? Problem Statement

$p(x)$, n → **Problem-intrinsic challenges**
Ambiguity among patterns of interest

$p(y|x)$ →
Variability and distortion
Noise in the environment

Problem-solver-intrinsic challenges
Lossy and faulty components

R_x, R_y →
Data storage limitations
Data processing limitations
Data transmission limitations

$R_c = \log M_c$ → **Operational challenges**
Number of objects to recognize

P_e^n → Reliability

Determine the admissible rates R_x, R_y, and R_c for reliable recognition

Information Theoretic View

- provides fundamental and generic understanding of compressive recognition problems.
- determines what is possible and what is impossible.
- guides designers, policy makers, and managers about resource allocation issues.

Coding Theoretic View

- provides applicable and specific approaches of compressive recognition problems.
- searches for what is practically possible.
- connects compressive recognition system design to rich results available in coding theory.

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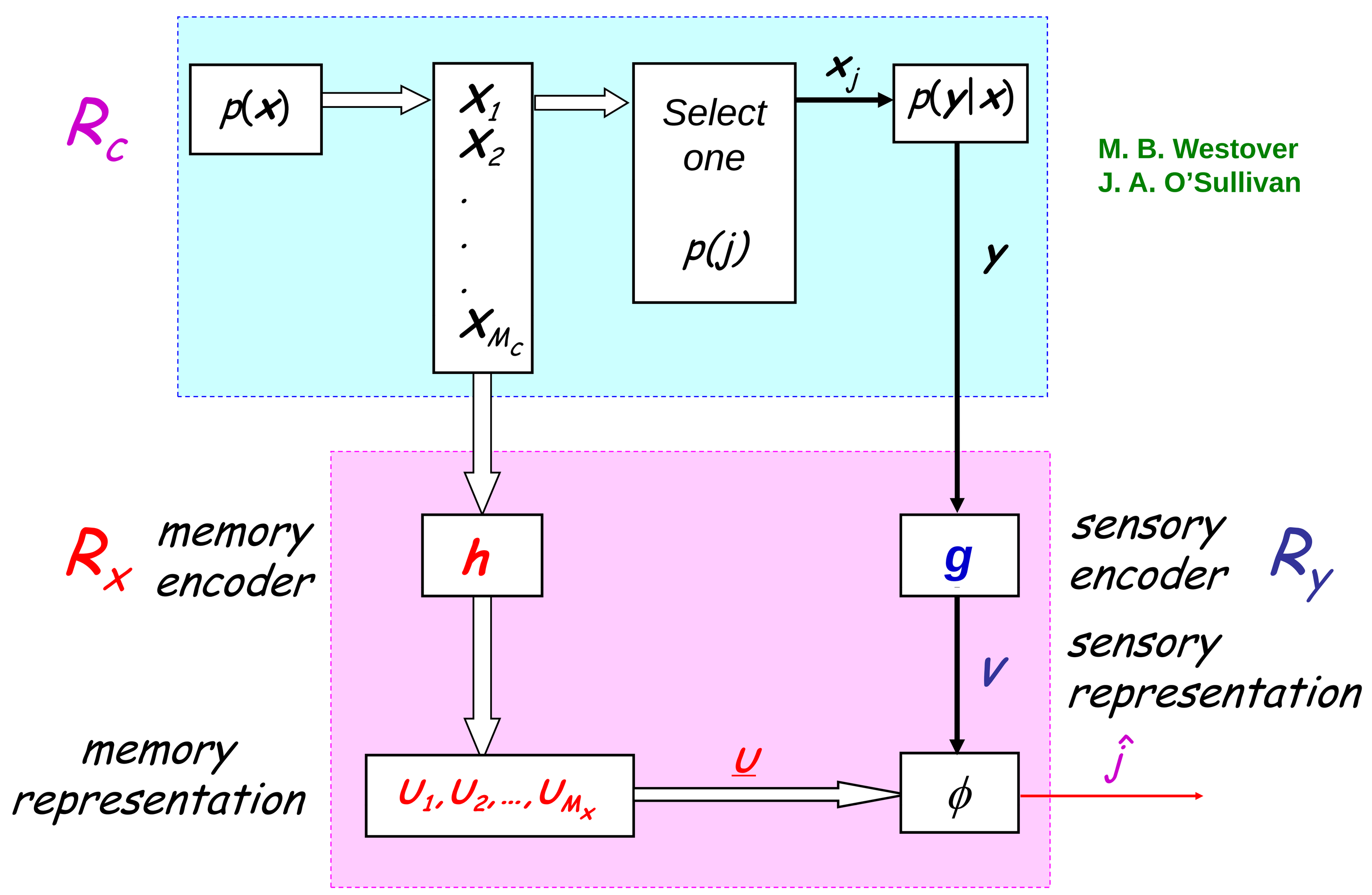


Objective: $\Pr\{\phi(V, \underline{U}) = j\} > 1-\varepsilon$, s.t. $R = (R_c, R_x, R_y)$

Definition. *An (M_x, M_y, M_c, n)* ***Pattern recognition system with linear encoding*** *for a given source $p(\mathbf{x})$ and observation channel $p(\mathbf{y}|\mathbf{x})$ consists of*

memory and sensory encoders based on linear code check matrices H and G that

$h(\mathbf{x}_i) = H\mathbf{x}_i = s_i$

$g(\mathbf{y}) = G\mathbf{y} = \sigma$

H and G can be
•LDPC parity check matrices,
•truncation matrix, or
•parity check matrices of other linear codes

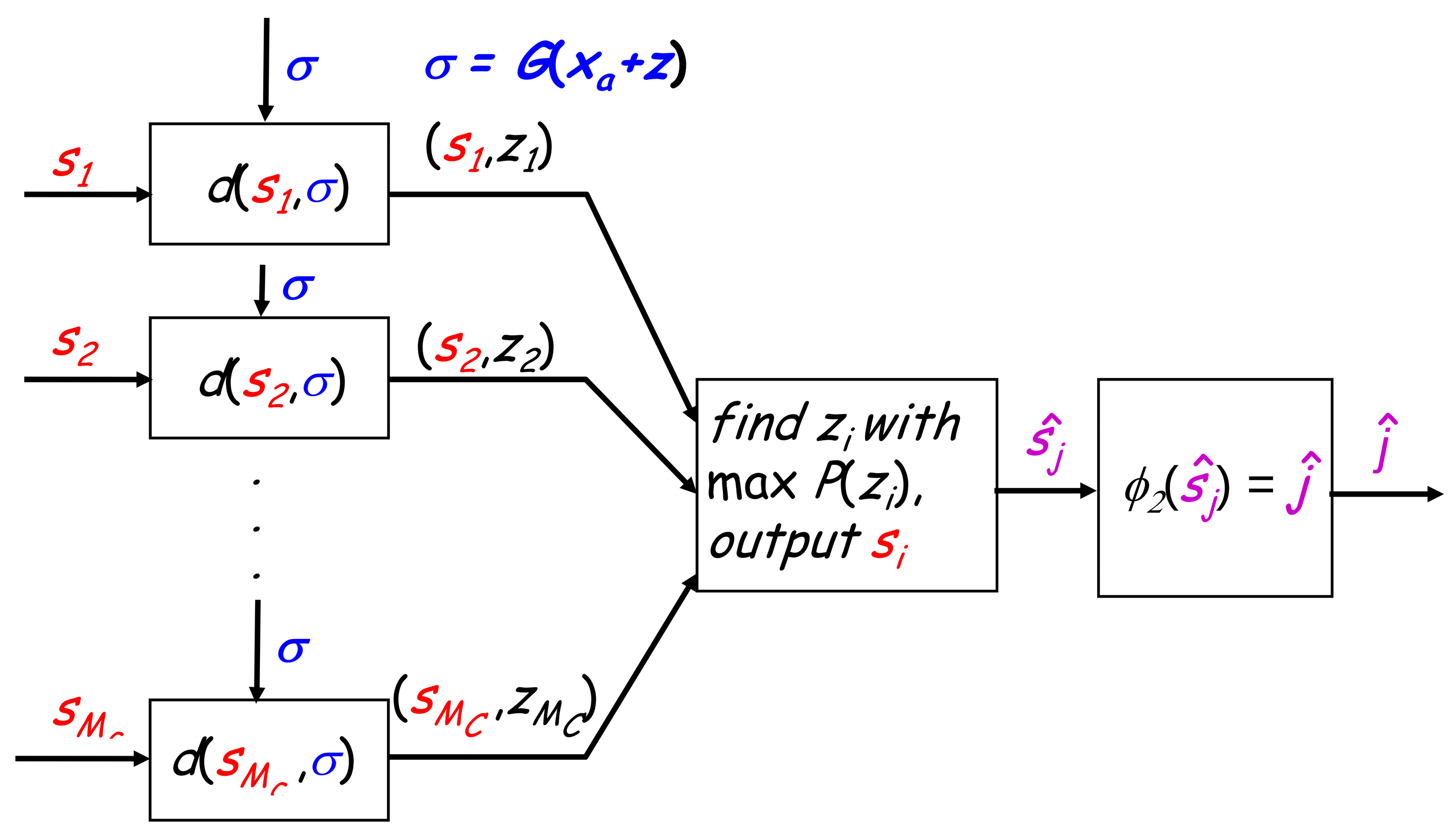
a recognition algorithm

$\phi(\sigma) = j$

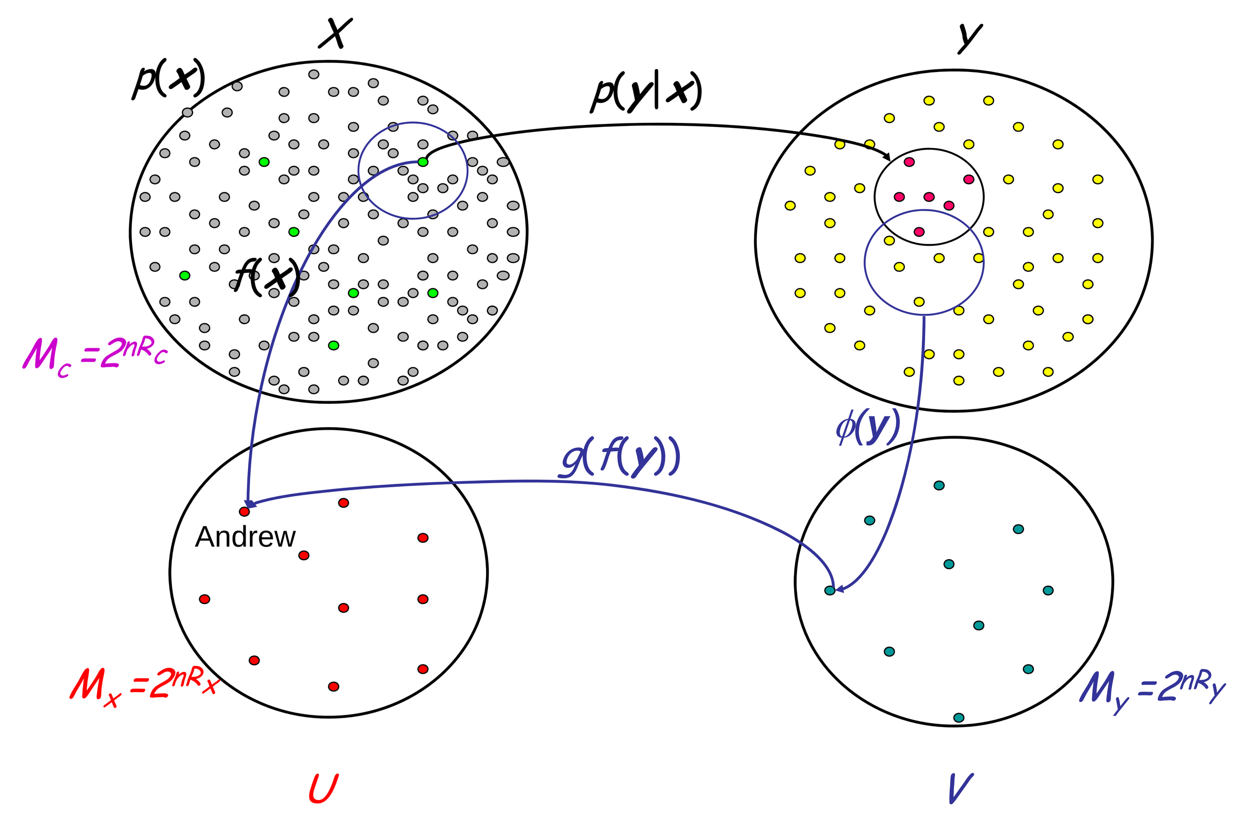
where ϕ consists of two algorithms (ϕ_1, ϕ_2) that

$\phi_1(\sigma) = s_j$

$\phi_2(s_j) = j$



$d(s_i, \sigma)$: decoding algorithm for linear code H
s_i: memory data
σ: sensory input



Theorem 1: Bounds on the achievable rate region

Inner bound: R^* is the set of rates (R_c, R_x, R_y) s.t. there exist random variables (U, V) satisfying:

1. $U - X - Y - V$
2. $R_x \geq I(X;U)$
   $R_y \geq I(Y;V)$
   $R_c \leq I(U;V) - I(U;V|X;Y)$

Inner bound: R^{**} is the set of rates (R_c, R_x, R_y) s.t. there exist random variables (U, V) satisfying:

1. $U - X - Y$, $X - Y - V$
2. $R_x \geq I(X;U)$
   $R_y \geq I(Y;V)$
   $R_c \leq I(U;V) - I(U;V|X;Y)$

M. B. Westover and J. A. O'Sullivan, 2008

Theorem 2: Toward the duality of compressive recognition design and communication code design

For each element of patterns drawn i.i.d. and uniformly from same finite set, if there exists a *good* ensemble of linear codes of rate $R = min(R_x, R_y)$ and a decoding algorithm for a noise distribution with entropy nR_z, Then for all

$R_c < \min(R_x, R_y) - R_z$,

there exists a *good* pattern recognition system design using the parity check matrix of the linear code for compression and the decoding algorithm for recognition.

Theorem 3: Truncation encoding and time sharing

For patterns and additive noise where elements are drawn i.i.d. from distributions Q_x and Q_z over some finite field, the P_e^n of truncation encoding goes to zero as n goes to infinity if

$R_c < \min(R_x, R_y)(H(Q_x * Q_z) - H(Q_z))$

Summary

- Good linear codes can be used as the basis for good recognition system design for general noise models.
- Recognition system design can be approached from designing linear codes for communication systems with the same channel noise model .
- Gaps remain among information theoretic bounds and recognition system design based on linear codes.

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1. P.-H Lai and J.A. O'Sullivan, "Pattern recognition system design with linear encoding for discrete patterns," ISIT 2007 Nice, France.
2. J.A. O'Sullivan and P.-H Lai, "Pattern recognition system design based on LDPC matrices," ISIT 2005 Adelaide, Australia.
3. M. B. Westover and J.A. O'Sullivan, "Achievable rates for pattern recognition," IEEE Trans. Info. Theory, Vol. 54, pp. 299- 320, 2008.