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SECOND-ORDER CONSENSUS OVER DIRECTED RANDOM NETWORKS

QIAN WANG (P.I.), JING ZHOU

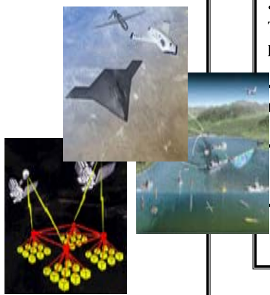
Mechanical Engineering, Penn State University, University Park, PA 16802

BACKGROUND

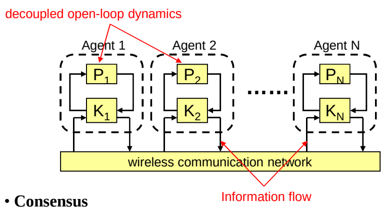
• **Coordinated Control**
A group of multiple (mobile) agents work cooperatively to achieve a common task.

• **Multi-agent System VS Single-agent System**
Robustness, feasibility, flexibility, cost, efficiency, etc.

• **Applications**
Multiple mobile robots
Clusters of telescope and satellite
Intelligent air traffic management
Autonomous underwater vehicles
Space formation flying
Unmanned aerial vehicles
Distributed sensor networks
Automated highways
Molecular conformation



• **Networked Multi-agent System**
Decoupled individual dynamics
Connected by information flow

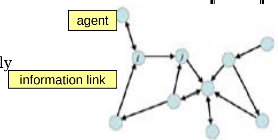


• **Consensus**
A group of agents reach an agreement on a certain quantity of interest which depends on the state of all agents.

PRELIMINARIES

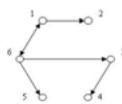
• **Graphs**
It is common to model information networks using graphs.

• **Random Graphs**
The existence of an edge between a pair of nodes is determined randomly and independently of other edges.



• **Adjacency Matrix**

• **Laplacian Matrix**



$$A=\begin{bmatrix}0&0&0&0&0&1\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&1&0&0&0\\0&0&0&0&0&1\\1&0&0&0&0&0\end{bmatrix}\qquad L=\begin{bmatrix}1&0&0&0&0&-1\\-1&1&0&0&0&0\\0&0&1&0&0&-1\\0&0&-1&1&0&0\\0&0&0&0&1&-1\\-1&0&0&0&0&1\end{bmatrix}$$

row sums equal 0

PROBLEM FORMULATION

• Consider n agents with identical open-loop dynamics

$$\dot{x}_{i,p}=x_{i,v},\ \dot{x}_{i,v}=u_i,\ i=1,\dots,n$$

• Agreement set
The agreement set $\mathscr{A}\subseteq\mathbf{R}^n$ is the subspace spanned by $\{\mathbf{1}\}$, i.e., $\{c\mathbf{1}\}$.

• Assumption 1 (Dynamically switching, directed, random networks)
The associated graph Laplacian is time-invariant during each sample period $[k\cdot\Delta t,(k+1)\cdot\Delta t), k=0,1,\dots\dots$

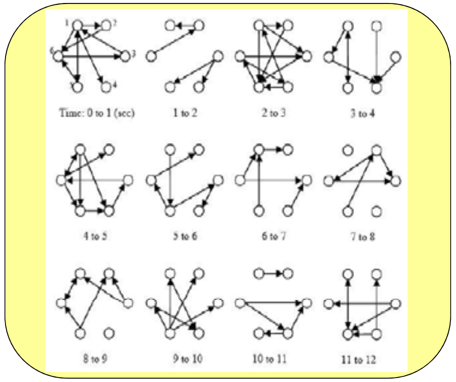
• Assumption 2: The directed graph associated with the expectation of the random Laplacian matrix, i.e., E[L], has a spanning tree.

• Asymptotic convergence to the agreement set with probability one

$$\mathrm{P}\{\sup_{k\ge N}\|x_k-c\cdot 1\|\ge\varepsilon\}\to 0,\ \text{as } N\to\infty.$$

• **Control objective**

$$\lim_{t\to\infty}x_p(k)=c_p\cdot 1,\ \text{w.p.1.}$$
$$\lim_{t\to\infty}x_v(k)=c_v\cdot 1,\ \text{w.p.1.}$$



• Directed graphs with unity edge weights
• Directed graphs with arbitrary positive edge weights
Directed graphs with arbitrary edge weights that are not necessary o be positive
Agents with decoupled second-order dynamics
Agents with dynamics that can be transformed into double ntegrators

APPROACHES

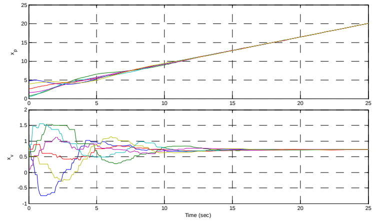
• **Consensus Algorithm**

$$u_i=\sum_{j\in N(i,t)}w_{ij}(\beta(x_{jp}-x_{ip})+\gamma(x_{jv}-x_{iv}))$$

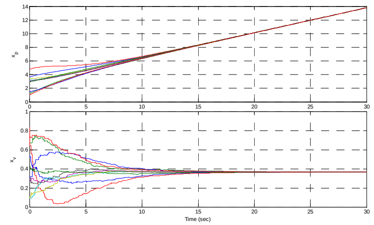
• **Convergence Proof**
Stochastic version of Lyapunov theory

SIMULATION RESULTS

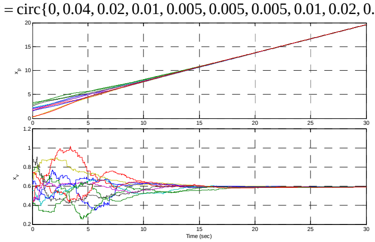
Case One (unity edge weights): $n=6,\ \Delta=0.1,\ \beta=\gamma=1,\ p=0.9$



Case Two (arbitrary positive edge weights):
$n=10,\ \Delta=0.01,\ \beta=1,\ \gamma=5,\ \mathrm{W}=\mathrm{circ}\{0,2,2,2,1,1,1,1,1,1\}$
$\mathrm{P}=\mathrm{circ}\{0,0.04,0.02,0.01,0.005,0.005,0.005,0.01,0.02,0.04\}$.



Case Three (arbitrary edge weights that are not necessary to be positive):
$n=10,\ \Delta=0.01,\ \beta=1,\ \gamma=5,\ \mathrm{W}=\mathrm{circ}\{0,2,2,2,1,1,1,-1,-1,-1\}$
$\mathrm{P}=\mathrm{circ}\{0,0.04,0.02,0.01,0.005,0.005,0.005,0.01,0.02,0.04\}$.



CONTRIBUTIONS

We have proved the convergence of a consensus algorithm for a group of agents with second-order open-loop dynamics over directed random information networks.

ACKNOWLEDGEMENT

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