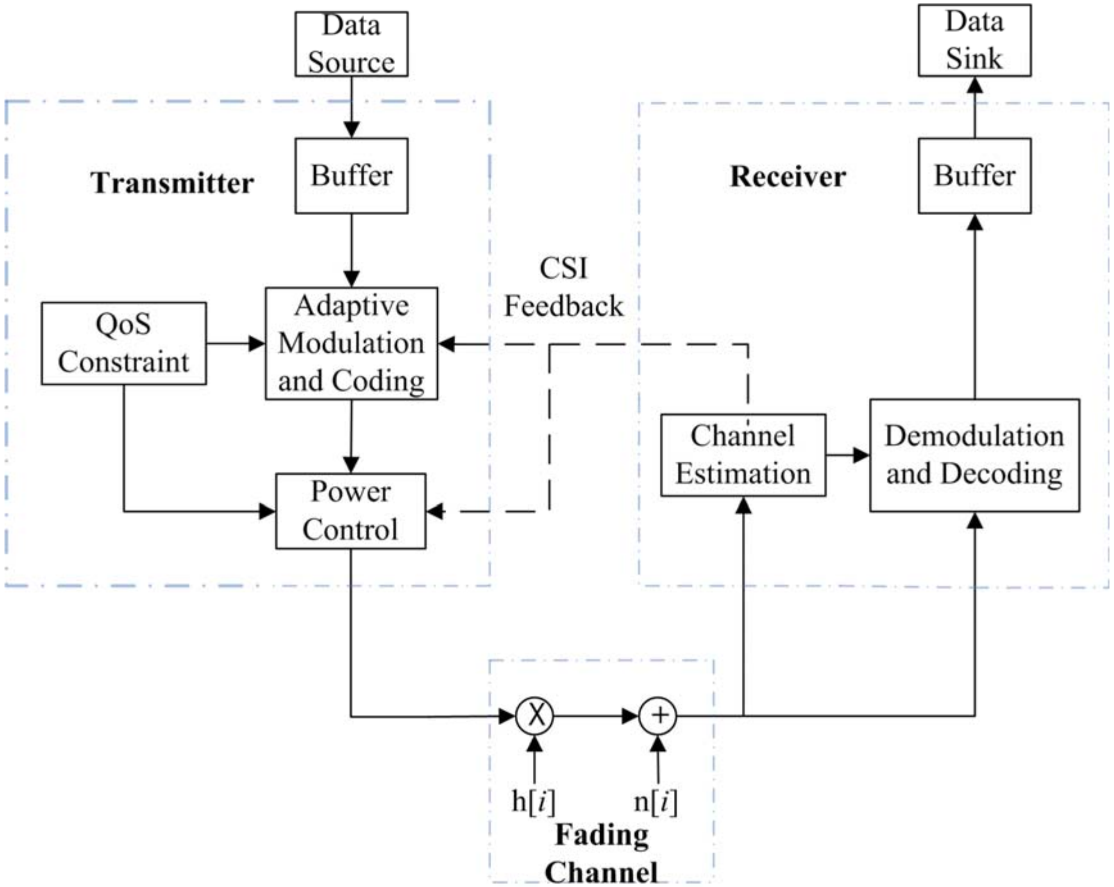
Analysis of Energy Efficiency in Fading Channels under QoS Constraints

Deli Qiao,Mustafa Cenk Gursoy, Senem Velipasalar
Department of Electrical Engineering,
University of Nebraska-Lincoln

Introduction

- Considerations for wireless systems
  - – Spectral efficiency.
  - – Energy efficiency.
  - – Various QoS guarantees, e.g., delay in real-time services.
- Useful ideas
  - – Spectral efficiency-bit energy tradeoff.
  - – Statistical QoS guarantees and the effective capacity.

System Model



System Model

- Simplified form

$$y[i] = h[i]x[i] + n[i], \qquad i = 1, 2, \ldots$$

  - Average power constraint $\mathrm{E}\left\{|x[i]|^2\right\} \le \overline{P}$.
  - $n[i]$ is zero-mean, complex Gaussian random variable with variance N_0 .
  - Frame length T, bandwidth B , and hence signal-to-noise ratio $\mathrm{SNR} = \overline{\mathrm{P}} / (N_0 B)$.
- Instantaneous service rate $R[i]$
  - Only receiver has CSI: $R[i] = B \log_2 \left(1 + \mathrm{SNR} z[i]\right)$ bits/s.
  - Transmitter also has CSI: $R[i] = B \log_2 \left(1 + \mu_{\mathrm{opt}}(\theta, z[i]) z[i]\right)$ bits/s.
  - Above, $z[i] = |h[i]|^2$.

System Model

- Optimal power-adaptation policy

$$\mu_{opt}(\theta,z)=\begin{cases}\dfrac{1}{\alpha^{\frac{1}{\beta+1}}z^{\frac{\beta}{\beta+1}}}-\dfrac{1}{z}, & z\geq\alpha\\ 0 & z<\alpha\end{cases}.$$

  - α is the channel threshold.
  - $\beta=\frac{\theta TB}{\log_e 2}$ is the normalized QoS exponent.
  - Average power constraint $\mathrm{E}\left\{\mu_{\mathrm{opt}}(\theta,z)\right\}=\mathrm{SNR}$.

Effective Capacity

- Effective capacity
  - Delay violation probability can be expressed as $\Pr\left\{D(\infty) > D_{\max}\right\} \approx e^{-\theta D_{\max}}$.
  - Let $\left\{R[i], i = 1, 2, \ldots\right\}$ be the discrete-time stationary and ergodic stochastic service process and $S[t] \triangleq \sum_{i=1}^{t} R[i]$ be the time-accumulated process, the Gartner-Ellis limit of $S[t]$ is

$$\Lambda_C(\theta) = \lim_{t\to\infty} \frac{1}{t} \log_e \mathrm{E}\left\{e^{\theta S[t]}\right\}.$$

  - Effective capacity is defined as

$$C_E(\mathrm{SNR}, \theta) = -\frac{\Lambda_C(-\theta)}{\theta} \text{ bits/s.}$$

Spectral Efficiency-Bit Energy

- Spectral efficiency vs. bit energy
  - The spectral efficiency in bits/s/Hz is

$$\mathbf{C}_E(\mathrm{SNR},\theta) = \frac{C_E(\mathrm{SNR},\theta)}{B} = -\frac{1}{\theta TB}\log_e \mathrm{E}\left\{e^{-\theta TR[i]}\right\}.$$

  - We have shown that $\mathbf{C}_E(\mathrm{SNR})$ is a concave function of SNR.
  - Minimum bit energy for reliable communication is

$$\left.\frac{E_b}{N_0}\right._{\min} = \frac{1}{\dot{\mathbf{C}}_E(0)}.$$

  - The slope S_0 of spectral efficiency versus E_b/N_0 is

$$S_0 = -\frac{2\left(\dot{\mathbf{C}}_E(\mathrm{SNR})\right)^2}{\ddot{\mathbf{C}}_E(\mathrm{SNR})}\log_e 2.$$

Energy Efficiency in the Low-Power Regime

- We first consider the low-power regime and assume $\overline{P} \to 0$ while the bandwidth B is fixed.
- CSI at the receiver only
  - The spectral efficiency for given θ in terms of SNR is

$$\mathbf{C}_E(\mathrm{SNR}) = -\frac{1}{\theta TB}\log_e \mathrm{E}\left\{(1+\mathrm{SNR}z)^{-\beta}\right\} .$$

  - We have derived that

$$\left.\frac{E_b}{N_0}\right._{\min} = \frac{\log_e 2}{\mathrm{E}\{z\}} , \quad S_0 = \frac{2}{(\beta+1)\frac{\mathrm{E}\{z^2\}}{\left(\mathrm{E}\{z\}\right)^2} - \beta} .$$

  - Then, the minimum received bit energy is

$$\left.\frac{E_b^{\,r}}{N_0}\right._{\min} = \left.\frac{E_b}{N_0}\right._{\min} \mathrm{E}\{z\} = -1.59\mathrm{dB}.$$

Energy Efficiency in the Low-Power Regime

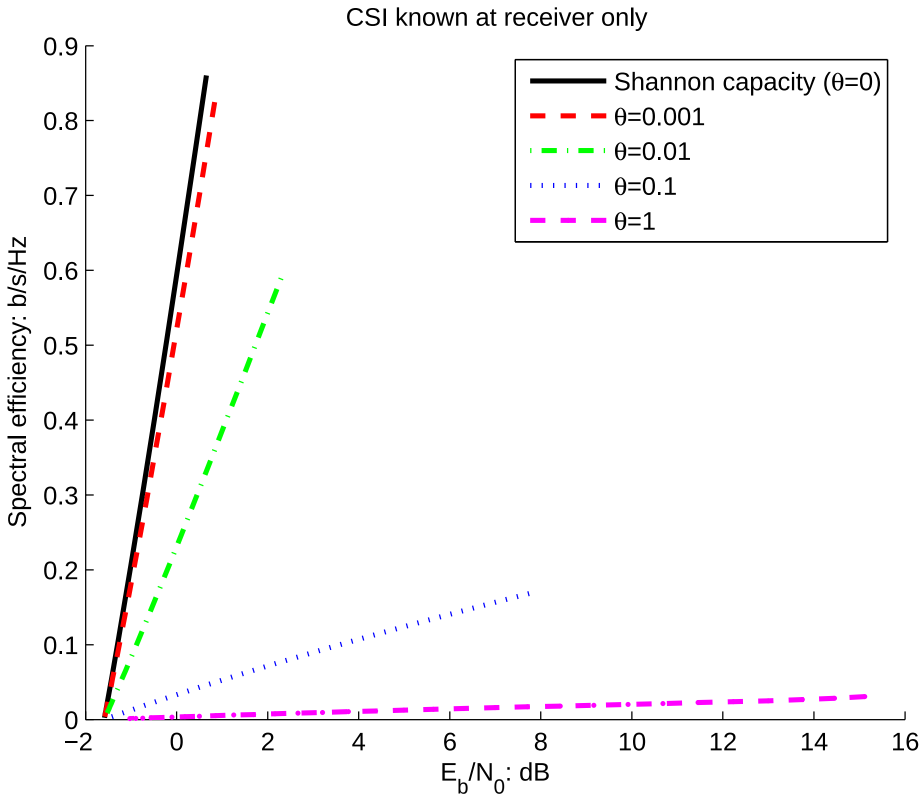


Fig.2 Spectral efficiency vs. E_b / N_0 with fixed B.

Energy Efficiency in the Low-Power Regime

- CSI at both the transmitter and receiver
  - The spectral efficiency for given θ in terms of SNR is

$$\mathbf{C}_E(\mathrm{SNR}) = -\frac{1}{\theta TB}\log_e\left(F(\alpha) + \mathrm{E}\left\{\left(\frac{z}{\alpha}\right)^{-\frac{\beta}{\beta+1}}\tau(\alpha)\right\}\right)$$

where $F(\alpha) = \mathrm{E}\{1\{z < \alpha\}\}$, $\tau(\alpha) = 1\{t \geq \alpha\}$.

  - We have shown that

$$\left.\frac{E_b}{N_0}\right._{\min} = \frac{\log_e 2}{z_{\max}}.$$

  - $z_{\max}$ is the maximum value that the random variable z can take, and for distributions with unbounded support, we have $\left.\frac{E_b}{N_0}\right._{\min} = 0$.

Energy Efficiency in the Low-Power Regime

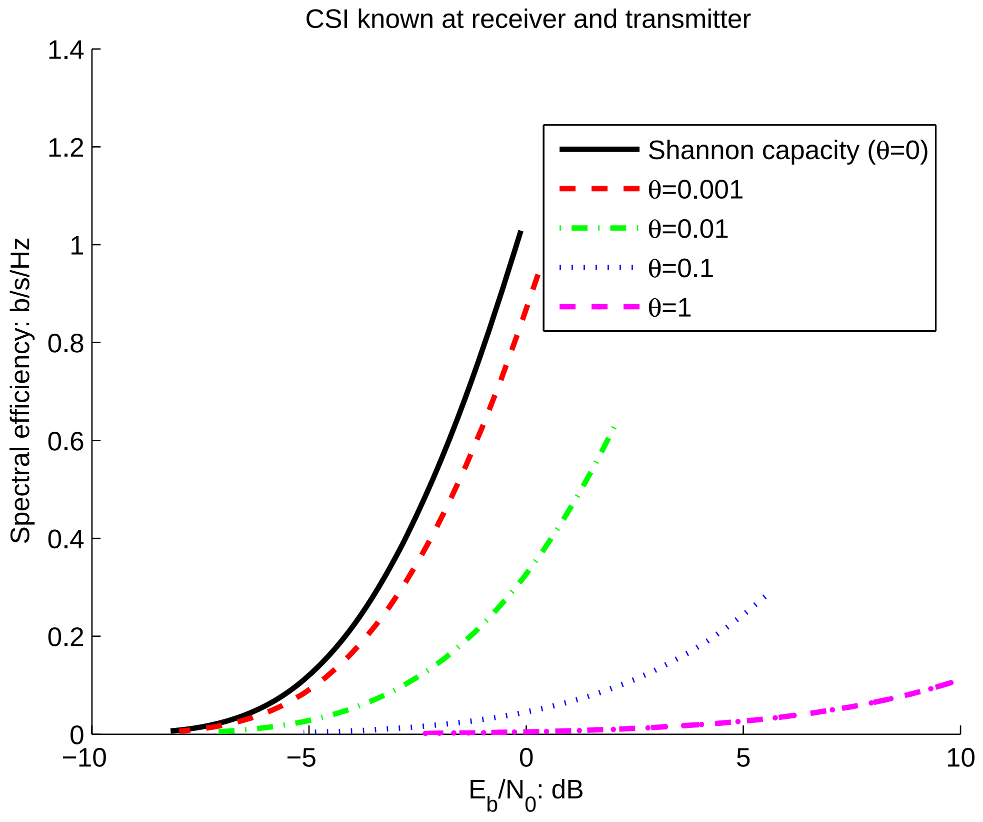


Fig. 3 Spectral efficiency vs. E_b / N_0 with fixed B.

Energy Efficiency in the Wideband Regime

- We study the performance at high bandwidths while the average power $\overline{P}$ is kept fixed.
- CSI at the receiver only
  - The spectral efficiency as a function of $\varsigma = \frac{1}{B}$ is:

$$\mathbf{C}_E(\varsigma) = -\frac{\varsigma}{\theta T} \log_e \mathrm{E}\left\{ e^{-\frac{\theta T}{\varsigma} \log_2\left(1 + \frac{\overline{P}\varsigma}{N_0} z\right)} \right\}.$$

  - We have shown that

$$\frac{E_b}{N_0}_{\min} = \frac{-\frac{\theta T \overline{P}}{N_0}}{\log_e \mathrm{E}\left\{ e^{-\frac{\theta T \overline{P}}{N_0 \log_e 2} z} \right\}} \geq \frac{\log_e 2}{\mathrm{E}\{z\}},$$

$$S_0 = 2\left(\frac{N_0 \log_e 2}{\theta T \overline{P}}\right)^2 \frac{\mathrm{E}\left\{ e^{-\frac{\theta T \overline{P}}{N_0 \log_e 2} z} \right\} \left(\log_e \mathrm{E}\left\{ e^{-\frac{\theta T \overline{P}}{N_0 \log_e 2} z} \right\} \right)^2}{\mathrm{E}\left\{ e^{-\frac{\theta T \overline{P}}{N_0 \log_e 2} z} z^2 \right\}}.$$

Energy Efficiency in the Wideband Regime

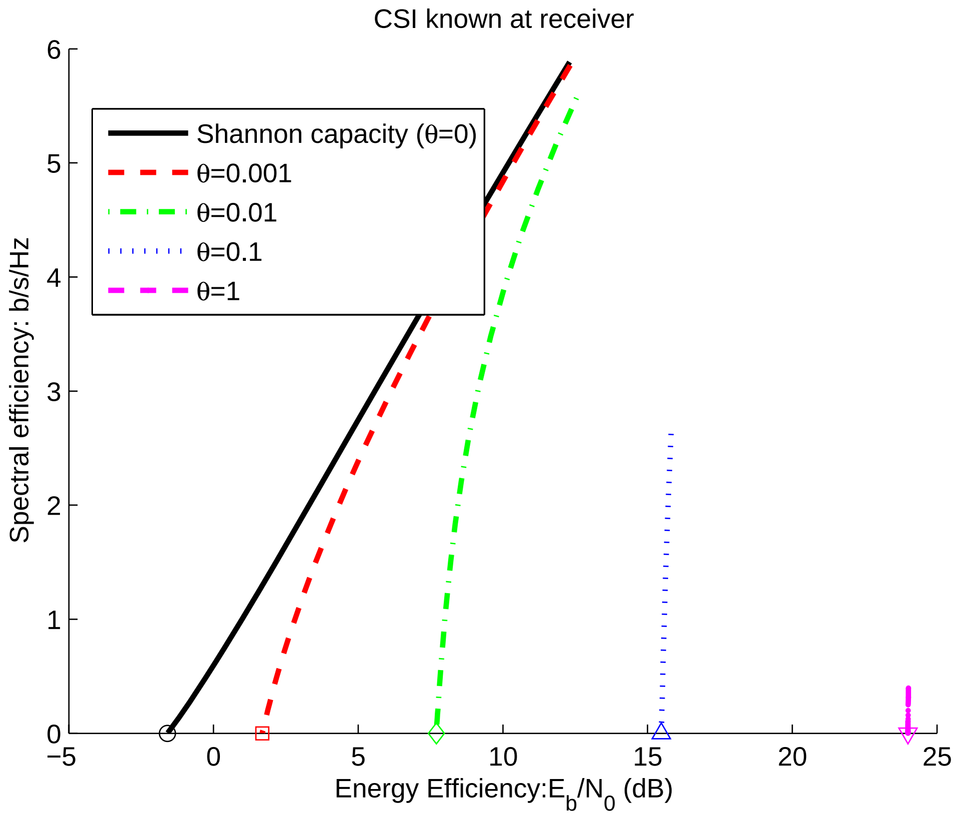


Fig. 4 Spectral efficiency vs. E_b/N_0 with fixed $\overline{P}$.

Energy Efficiency in the Wideband Regime

- CSI at both the transmitter and receiver
  - The spectral efficiency as a function of $\varsigma = 1/B$ is:

$$\mathbf{C}_E(\varsigma) = -\frac{\varsigma}{\theta T}\log_e\left(F\left(\alpha\right) + \mathrm{E}\left\{ \left(\frac{z}{\alpha}\right)^{-\frac{\theta T}{\theta T + \varsigma \log_e 2}} \tau(\alpha) \right\} \right).$$

  - In wideband regime, we have

$$\lim_{\varsigma \to 0} \alpha(\varsigma) = \alpha^*,$$

where α^* satisfies

$$\mathrm{E}\left\{ \left[\frac{\log_e(\frac{z}{\alpha^*})}{z} \right] \tau(\alpha^*) \right\} = \frac{\theta T \overline{P}}{N_0 \log_e 2}.$$

  - Then, we can show that

$$\frac{E_b}{N_0}_{\min} = -\frac{\theta T \overline{P}}{N_0 \log_e \xi}, \quad S_0 = \frac{\xi \left(\log_e \xi\right)^2 N_0 \log_e 2}{\theta T \overline{P} \alpha^*},$$

where $\xi = F(\alpha^*) + \mathrm{E}\left\{ \frac{\alpha^*}{z} \tau(\alpha^*) \right\}$.

Energy Efficiency in the Wideband Regime

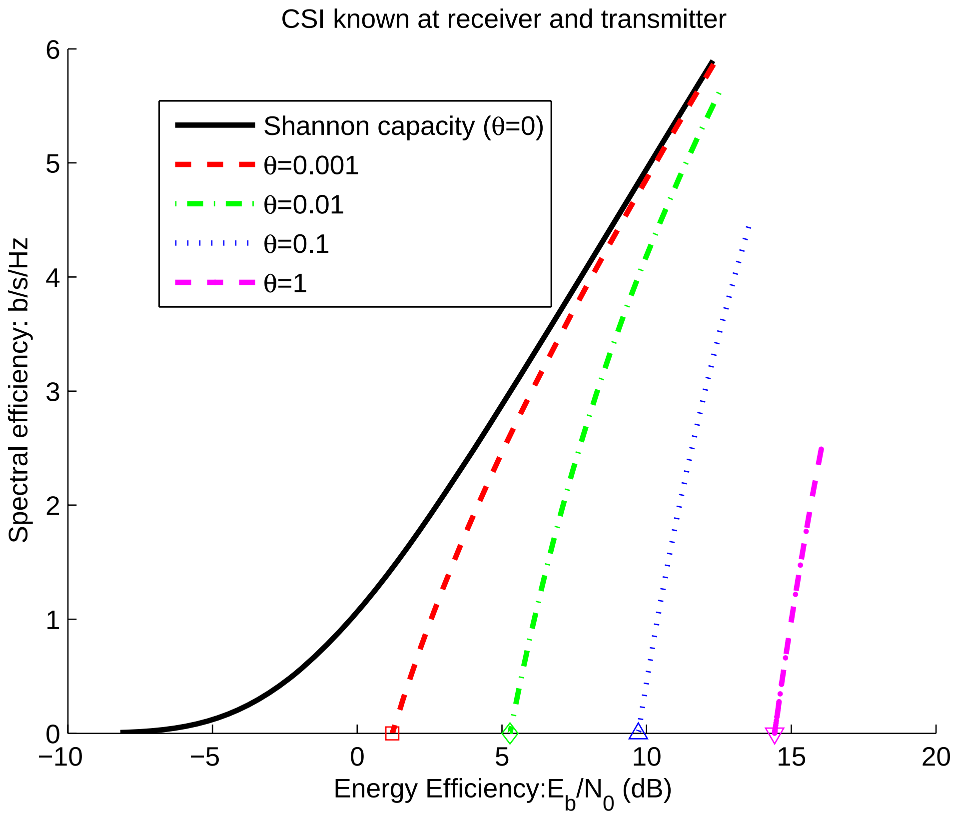


Fig. 5 Spectral efficiency vs. E_b / N_0 with fixed $\overline{P}$.

Conclusion

- We have characterized an energy-delay tradeoff by considering the effective capacity
  - Expressions for the minimum bit energy and wideband slope in the presence of QoS constraints have been obtained.
  - Through the analysis of spectral efficiency-bit energy tradeoff, we have quantified the increased energy requirements.
  - While the bit energy levels can approach in the low-power regime to those that can be obtained in the absence of QoS constraints, strictly higher bit energy values are needed in wideband regime.

References

- [1] T. S. Rappaport, "*Wireless Communications: Principles and Practice*", 2nd ed. Prentice Hall PTR, 2001
- [2] S. Verdu, "Spectral efficiency in the wideband regime," *IEEE Trans. Inform. Theory*, vol.48, no.6 pp.1319-1343. Jun.2002
- [3] D. Wu and R. Negi "Effective capacity: a wireless link model forsupport of quality of service," *IEEE Trans. Wireless Commun.*, vol.2,no.4, pp.630-643. July 2003
- [4] D. Wu and R. Negi, "Effective capacity-based quality of servicemeasures for wireless networks," *Broadband Networks,2004.BroadNets 2004. Proc. First International Conference on*. pp. 527-536
- [5] D. Wu and R. Negi, "Downlink scheduling in a cellular network for quality-of-service assurance," *IEEE Trans. Veh. Technol.*, vol.53, no.5,pp. 1547-1557, Sep., 2004
- [6] D. Wu and R. Negi, "Utilizing multiuser diversity for efficient support of quality of service over a fading channel," *IEEE Trans. Veh. Technol.*, vol.49, pp. 1073-1096, May 2003
- [7] J. Tang and X. Zhang, "Quality-of-Service Driven Power and Rate Adaptation over Wireless Links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp.3058-3068, Aug. 2007
- [8] J. Tang and X. Zhang, "Quality-of-Service Driven Power and RateAdaptation for Multichannel Communications over Wireless Links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp.4349-4360, Dec. 2007
- [9] C.-S. Chang, "Stability, queue length, and delay of deterministic and stochastic queuing networks," *IEEE Trans. Auto. Control*, vol. 39, no. 5, pp. 913-931, May 1994
- [10] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004
- [11] S. Shamai, S. Verdu, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. Inform. Theory*, vol. 47, no. 4, pp. 1302-1327, May 2001
- [12] S. Borade and L. Zheng, "Wideband Fading Channels with Feedback," 42nd Allerton Annual Conference on Communication, Control and Computing, October 2004