Effect of Delay on the Capacity of Relay Networks

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ABSTRACT

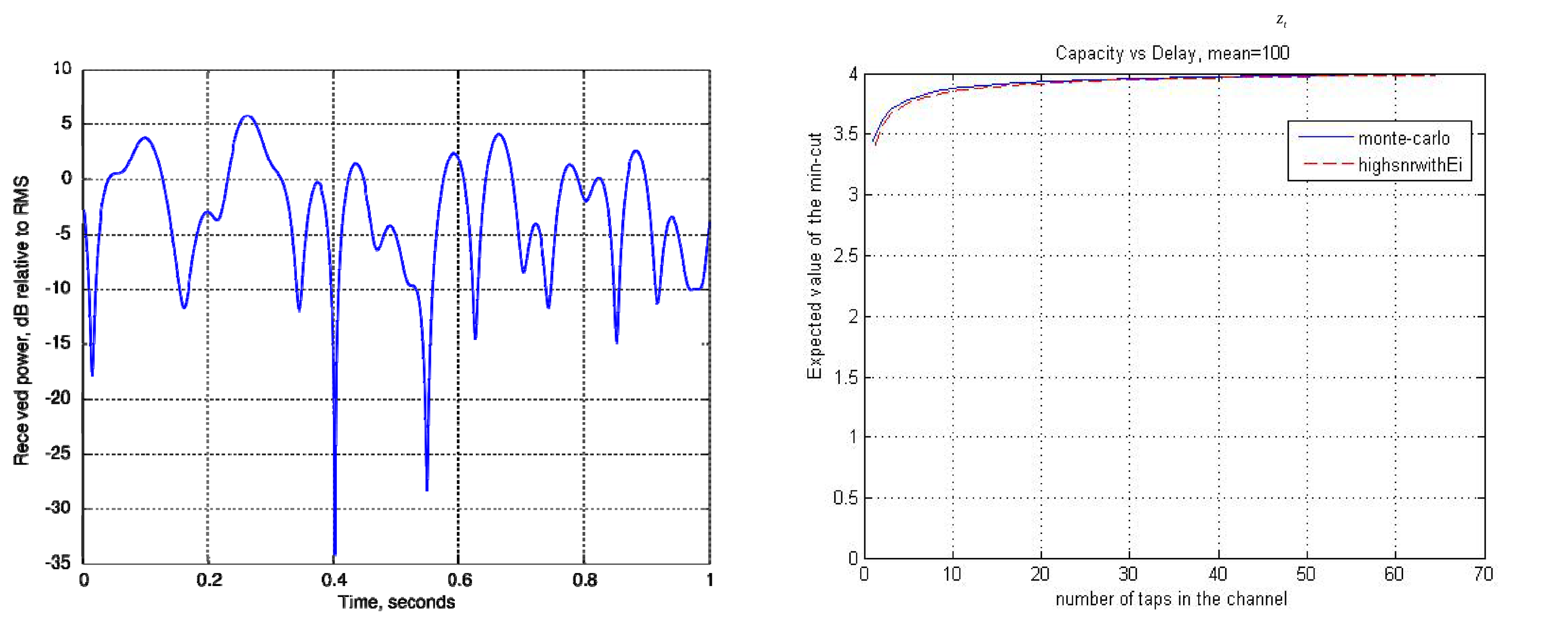
Network coding became an essential ingredient in achieving the capacity of a network. Linear network coding has been shown to be sufficient for achieving the max-flow bound of multicast networks. In our work, we investigated the delay effect on the average min-cut capacity of relay networks. The nodes in the network are considered to be in time-varying Rayleigh channels. We define the delay as the number of taps in the time-varying channel and study the effect of it on the capacity of the network. For different types of networks closed-form expressions of the min-cut capacity as a function of the number of taps are derived.

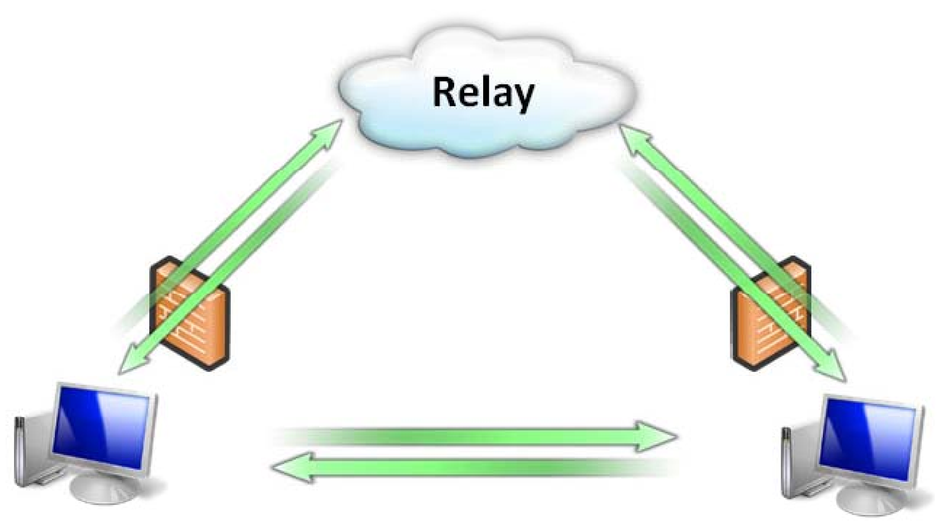
SYSTEM MODEL

- There are D number of channel taps during a transmission and the channels are assumed to be iid and subject to Rayleigh block fading channel.
- The capacity of a wireless link subject to block Rayleigh fading is given by: $C=1/D*\sum\log(1+|h_i|^2 SNR)$ where $|h_i|^2 \sim Exp(1)$.
- $E[C] = \exp(1/SNR)\, Ei(1,1/SNR)$
- $Var[C] \approx Var[1/D*\sum\log(|h_i|^2 SNR)] = \pi^2/6/D$

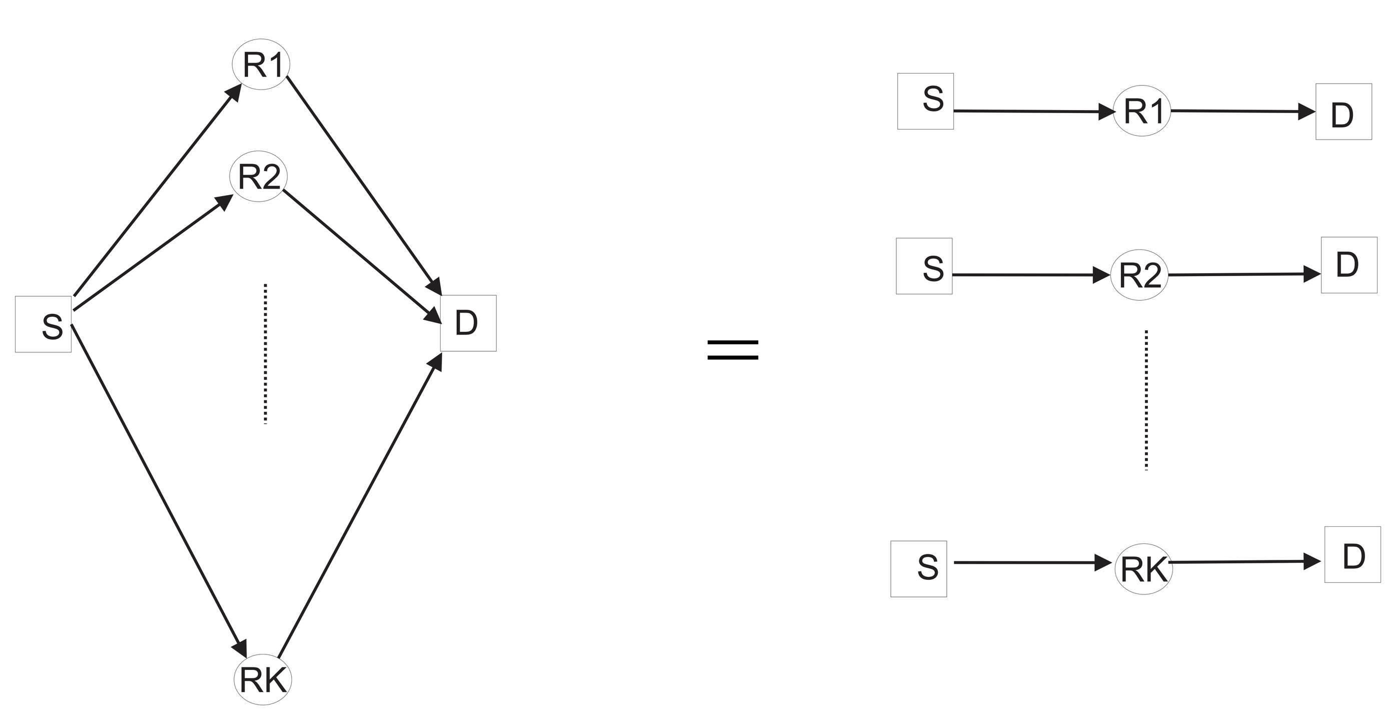
➢**GOAL:** We would like to find the relation between the number of taps D and the expected value of the min-cut capacity.

- **Lemma 1:** Let $W=\min(C_1,C_2)$. When C is approximated to be a Gaussian with mean E[C], Var[C], then $E[W] = E[C] - \text{sqrt}(\pi/6/D)$.





RELAY NETWORKS



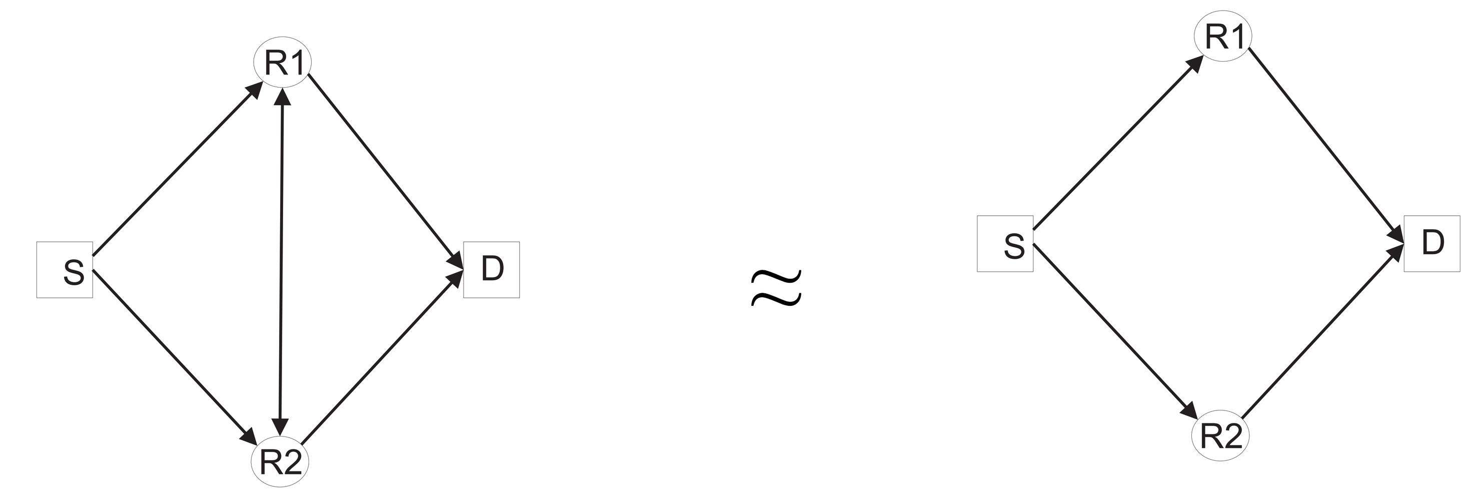
Proposition 1: The capacity of the single source, single destination and K relay network is equal to the sum of the capacities of the K single relay network. Hence $E[C_{nt}] = K * \{E[C] - \text{sqrt}(\pi/6/D)\}$

Proof: Consider two relay case. Let the capacity of the links be C_{SR1}, C_{SR2}, C_{R1D}, C_{R2D}. The min-cut of this network is :

$\text{Min}\{ C_{SR1}+C_{SR2}, C_{R1D} + C_{SR2}, C_{SR1} + C_{R2D}, C_{R1D} + C_{R2D}\}=\min\{\min\{ C_{SR1}+ C_{SR2}, C_{R1D} + C_{SR2}\}+\min\{ C_{SR1} + C_{R2D}, C_{R1D} + C_{R2D} \}\}=\min\{ C_{SR2+\min}\{ C_{SR1}, C_{R1D}\}, C_{R2D+\min}\{ C_{SR1}, C_{R1D} \}\}= \min\{ C_{SR1}, C_{R1D}\}+\min\{ C_{SR2}, C_{R2D}\}$

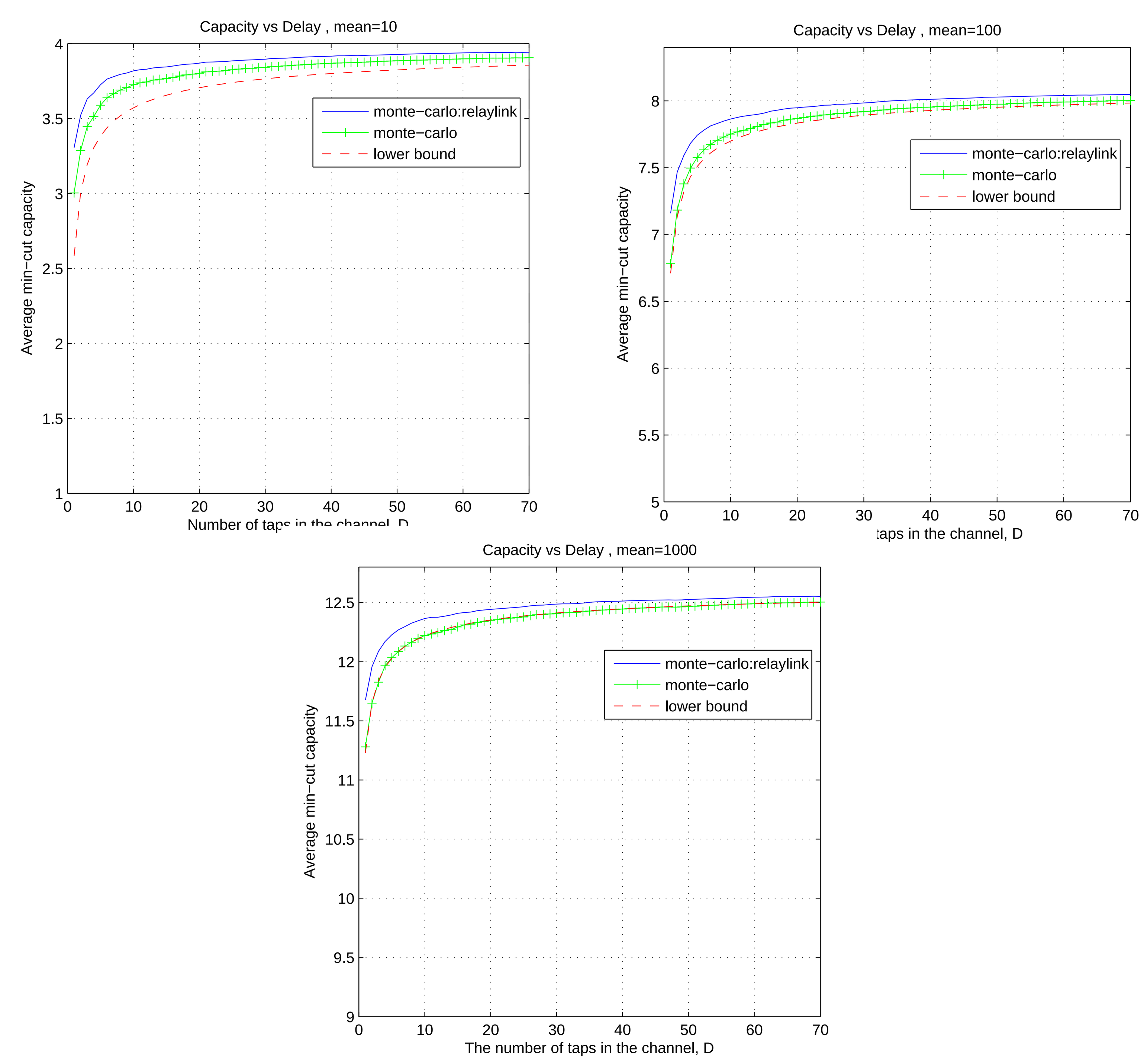
Now using Lemma 1 result follows.

Do We Really Need a link between the Relays?



Proof: Let the capacity of the links be C_{SR1}, C_{SR2}, C_{R1D}, C_{R2D}, $C_{R1R2} \sim N(\mu,\sigma^2)$. The min-cut of this network is $W= \text{Min}\{ A_1 , A_2 + C_{R1R2} \}$ where $A_1 = \min\{ C_{SR1}+ C_{SR2}, C_{R1D} + C_{R2D}\}$ and
$A_2 = \min\{ C_{SR1}+ C_{R2D}, C_{R1D} + C_{SR2}\}$. Then,
$E[W] = E[A_1| A_2 + C_{R1R2} - A_1>0]*P(A_2 + C_{R1R2} - A_1>0) + E[A_2+ C_{R1R2} | A_2 + C_{R1R2} - A_1<0]*P(A_2 + C_{R1R2} - A_1<0)$.
Now, letting $P(A_2 + C_{R1R2} - A_1<0) = \varepsilon$ we have $E[A_1| A_2 + C_{R1R2} - A_1>0] \approx E[A_1] \approx E[W]$.
$A_2 + C_{R1R2} - A_1 \sim N(\mu,3*\sigma2)$. Then, $P(A_2 + C_{R1R2} - A_1<0) = \varepsilon = Q(\mu/\sqrt{3}\sigma)$.
Hence, we need $\mu = Q^{-1} (\varepsilon)* \sqrt{3}\sigma$ in order to have a good approximation.

RESULTS & CONCLUSIONS



Conclusion: Expected value of the min-cut capacity in terms of the number of taps in the channel for different SNR values. Both analytical results and monte-carlo simulation results are given for the two networks above given in the figure. As can be seen from the figures, the link between the relays does not provide much contribution to the expected min-cut capacity.

- ❑To sum up, we have investigated how the number of channels in a Rayleigh fading environment effects the average min-cut capacity of a network which is an upper bound on the transmission capacity of the source node. Our results show that the decrease in the expected value of the capacity as a function of the number of taps in the channel is given by 1/sqrt(D) and reaches its expected value as the number of channels increases further by the law of large numbers.
- ❑Another interesting result was that we do not really need the link between two relays to increase the capacity of the transmission between the source and the destination.

OTHER ONGOING WORK and FUTURE WORK

- ❑ Studying the arbitrary type of networks
- ❑ Studying the non-iid links

Some Future Work

- ❑ Studying the effects of other parameters (such as number of hops or relays) other than the number of channels on the capacity of networks
- ❑ Studying the diversity