New Sequences of Capacity Achieving LDPC Code Ensembles over the Binary Erasure Channel

Hamid Saeedi and Amir H. Banihashemi
Carleton University, Ottawa, ON, Canada

- ***Capacity achieving sequences*:**
  For BEC, a sequence of degree distributions is called *capacity achieving with rate R* if the threshold can be made arbitrarily close to 1-*R* for sufficiently large maximum variable node and check node degrees.

- ***Existing sequences proposed by Shokorllahi:*** Tornado and right-regular sequences

- ***Decoding complexity for a given ensemble:***
  directly depends on average check node degree.

- ***Speed of convergence:*** for a capacity achieving sequence, it is very important to see that how fast it achieves the capacity as average check node degree increases.

- ***Asymptotically quasi-optimal sequences:***

  A sequence is called a*symptotically quasi-optimal* if its decoding complexity per iteration increases only logarithmically with the relative increase in the performance,

$$\mu(\lambda_n, \rho_n) = \frac{\overline{d}_c \log(R)}{\log(1 - \frac{\varepsilon}{1-R})} \qquad \lim_{n \to \infty} Sup(\mu(\lambda_n, \rho_n)) = \mu$$

- ***Asymptotically optimal sequences (μ = 1):***

  Only check regular sequences are a*symptotically optimal.*

$$\Delta(\lambda^n, \rho^n) = \frac{\Im(\lambda^n, \rho^n)}{R^{\overline{d}_c}} \qquad \lim_{n \to \infty} Sup \Delta(\lambda^n, \rho^n) = \Delta$$

- Problems with Shokrollahi's sequences:

- For a given check node degree distribution, all constituent variable node degrees from 2 to maximum variable node degree N are present. ***There is no control over the number of constituent variable node degree (P), i.e.; P= N – 1.***

- The only asymptotically optimal sequences are check regular sequences. In these sequences, for a given d_c, ***there is no control on the maximum variable node degree.***

- If we want to have smaller D_v for a given d_c, we have to use other types of sequences such as Tornado sequences. However, any sequence other than check regular sequences, is not asymptotically optimal meaning that ***maximum variable node degree is decreased at the expense of performance degradation.***

- Our proposed sequences is able to address the problems associated with Shokrollahi's sequences.
- ***Construction***:

$$\rho^{-1}(1-x) = 1 - \sum_{i=2}^{\infty} T_i x^{i-1}, T_i > 0$$

$$T_i = \binom{\alpha}{i-1}(-1)^i, \alpha = 1/(D_c - 1)$$

- We determine *N* such that:

$$\frac{\sum_{i=2}^{N-1} T_i / i}{\sum_{i=2}^{N-1} T_i} \geq (1-R)\bar{d}_c^{-1} > \frac{\sum_{i=2}^{N} T_i / i}{\sum_{i=2}^{N} T_i}$$

- *GR* ensembles:

$$\varepsilon = \frac{\sum_{i=2}^{f(N)} T_i (1/i - 1/N)}{(1-R)^{-1}\bar{d}_c^{\,-1} - 1/N}, \quad \lambda_i = T_i / \varepsilon, 2 < i \le f(N); \lambda_N = 1 - \sum_{i=2}^{f(N)} \lambda_i .$$

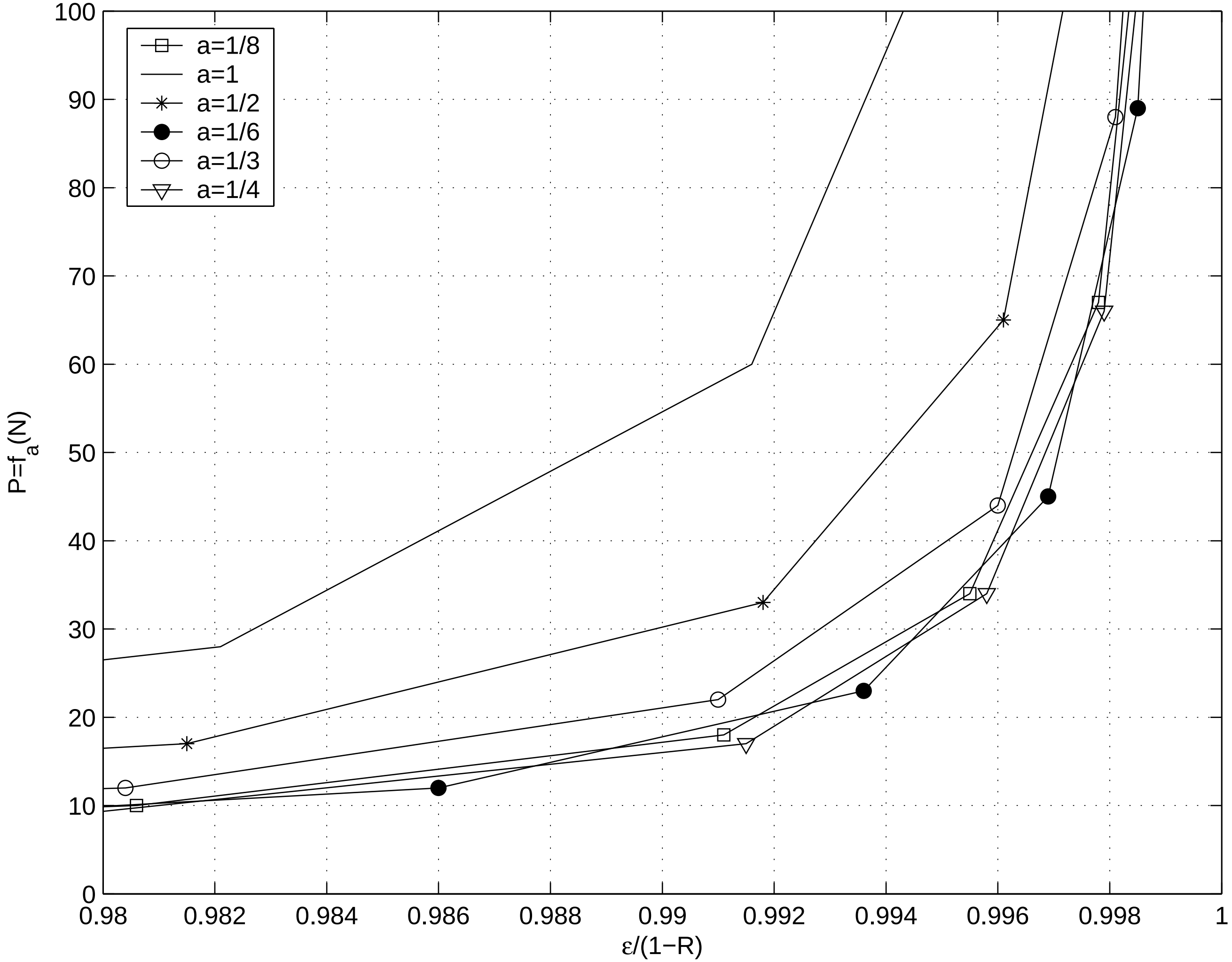
- *f*(*N*) is a strictly increasing function of *N*.
- *f*(*N*) =*N* -1 would construct a Shokrollahi's check regular ensemble sequence.

- *MGR* ensembles: replace *N* with D_v where D_v is the smallest number for which the resulting ensemble is convergent.

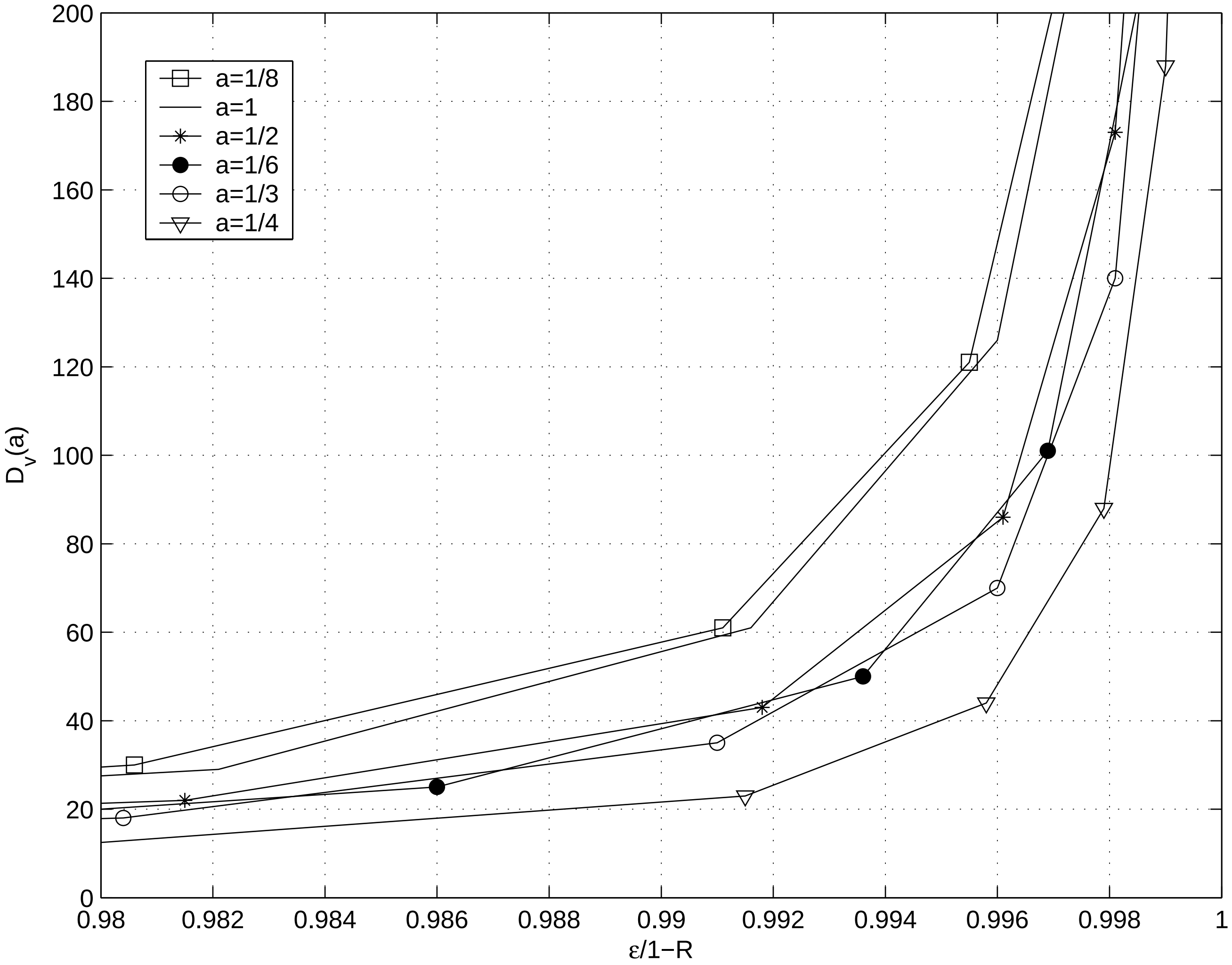
- *Theorem 3*: Our ensemble sequences achieve the capacity as D_c tends to infinity.

- *Theorem 4*: If the function $f(N)$ is chosen such that $\lim_{\alpha \to 0} f(N)^{\alpha} = M$ where $1 < M < 1/R$, the resulting sequence is *asymptotically quasi-optimal* with constant $\mu = -\ln R / \ln M$.

- *Theorem 5*: If the function $f(N)$ is chosen such that $\lim_{\alpha \to 0} \frac{f(N)}{N} = K > 0$, the resulting sequence is *asymptotically optimal* with $\Delta = e^{\gamma}(1/K - \ln(1/K))$ where $\gamma = .577215$.



The required number of constituent variable node degrees versus the performance of rate-1/2 *MGR* ensembles with *fa*(*N*) = *aN* +*b* for different values of *a*.



The required maximum variable node degree versus the performance of rate-1/2 *MGR* ensembles with $f_a(N) = aN + b$ for different values of a.

- To be presented in ISIT 2008.

- A complete version:

H. Saeedi, A. Banihashemi, “New Sequences of Capacity Achieving LDPC Code Ensembles over the Binary Erasure Channel,”, Submitted to Trans. Information Theory, Jan 2008, a revision to be submitted in June. 2008.