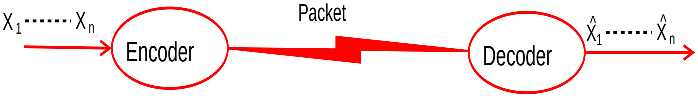
Multiple Descriptions with Feed-forward

Ramji Venkataramanan
Dept. of EECS, University of Michigan

joint work with Prof. Sandeep Pradhan

Multiple Descriptions



- Source X: compressed into packets
- Packets may be dropped
- Compress X into two *different* packets

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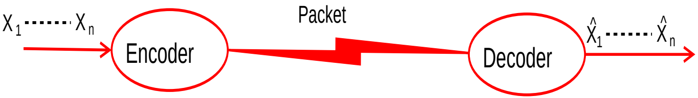

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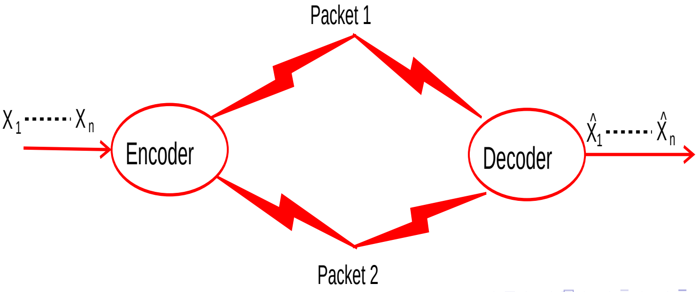

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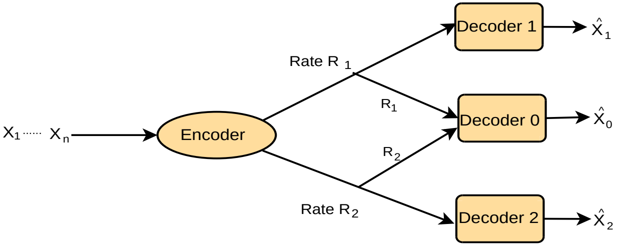

- Rate R_1 yields reconstruction $\hat{X}_1$ with distortion D_1
- Rate R_2 yields reconstruction $\hat{X}_2$ with distortion D_2

> - Want better quality D_0 if both packets received
> - $\hat{X}_1$ and $\hat{X}_2$ need to *refine* each other!

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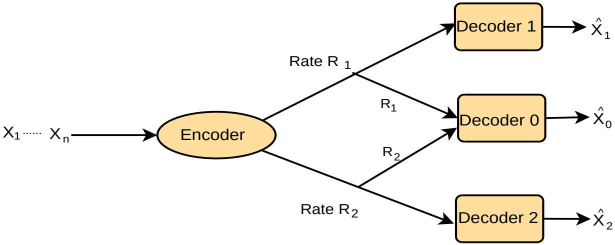

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GOAL

Rate	Distortion
R_1 bits/sample	D_1
R_2 bits/sample	D_2
$R_1 + R_2$ bits/sample	D_0

GOAL

Given i.i.d source P_X:

Find all achievable $(R_1, R_2, D_1, D_2, D_0)$

- Still an open problem
- Studied by [Cover, El Gamal], [Ahlswede], [Zhang, Berger], . . .
- Best known rate-region: [Zhang, Berger '87]

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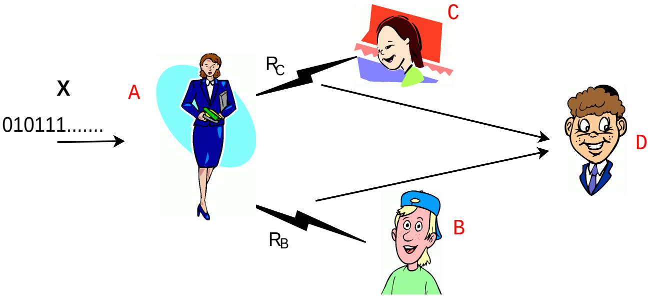
Example

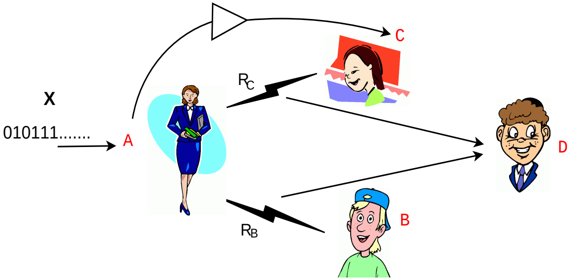


- Alice has an i.i.d binary source $\sim$ Bernoulli(1/2)
- Bob and Carol: distortion d using R_B, R_C bits/sample
- Dave gets Bob's bits and Carol's bits- needs to reconstruct perfectly!

Characterize

$r_{sum}(d) \triangleq$ Smallest sum-rate $R_B + R_C$ for distortion $(d, d, 0)$

Example with feed-forward



Same model as before, one extra feature...

- After Carol reconstructs each sample, Alice tells her if she made an error or not. *Feed-forward*
- Before reconstructing each sample, Carol knows *past* source samples

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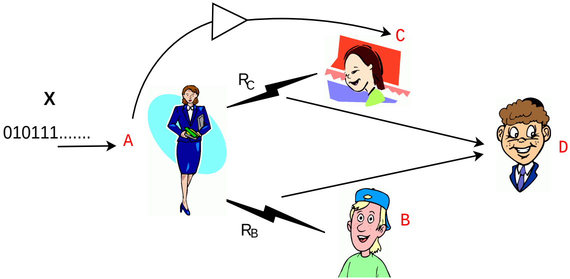

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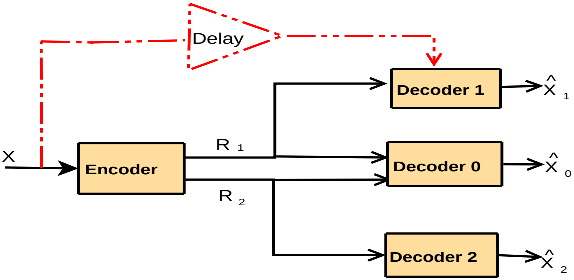
General feed-forward model



- Encoder mappings: $e_m : \mathcal{X}^N \to \{1, \ldots, 2^{NR_m}\}, \quad m = 1, 2.$
- Mappings for decoders 2 and 0:

$$g_2 : \{1, \ldots, 2^{NR_2}\} \to \widehat{\mathcal{X}}_2^N$$

$$g_0 : \{1, \ldots, 2^{NR_1}\} \times \{1, \ldots, 2^{NR_2}\} \to \widehat{\mathcal{X}}_0^N$$

- A sequence of mappings for decoder 2:

$$g_{1n} : \{1, \ldots, 2^{NR_1}\} \times \mathcal{X}^{n-k} \to \widehat{\mathcal{X}}_1, \quad n = 1, \ldots, N.$$

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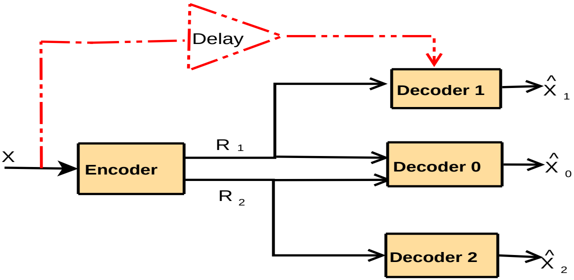

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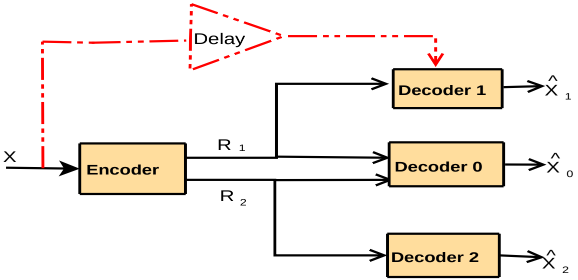

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- Single-letter achievable region for MD with FF (ISIT '08)
- Better than best no-FF region [Zhang,Berger '87]
- FF helps even for an i.i.d source

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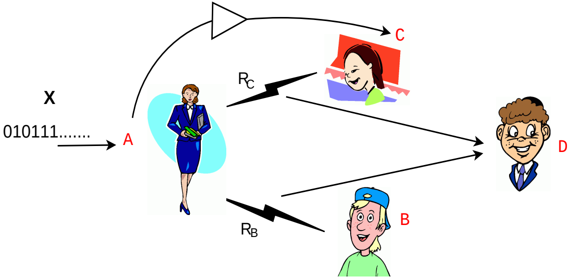
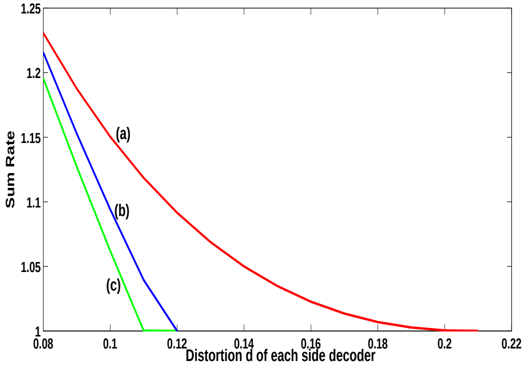


Comparison



- (a): Lower bound without feed-forward [Zhang, Berger '87]
- (b): Achievable sum-rate with FF- better than optimal w/o FF
- (c): Rate-dist lower bound with FF- $R_B + R_C > 2(1 - h(d))$

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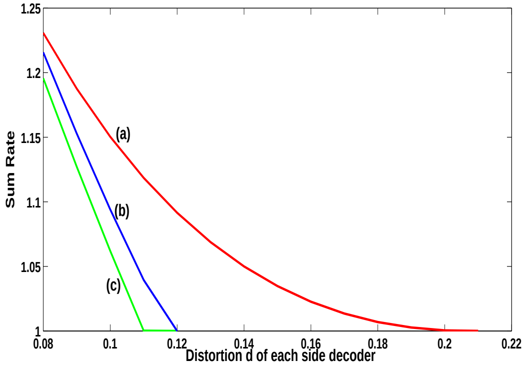

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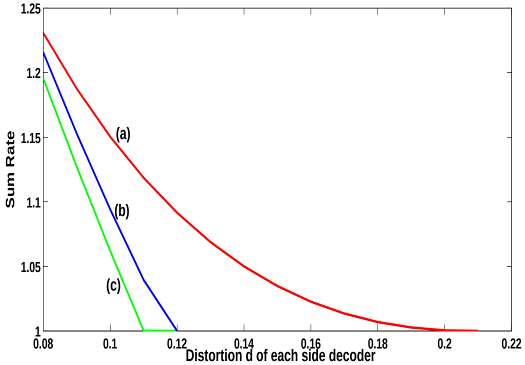

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