A New Upper Bound for a Binary Additive Multiple Access Channel with Feedback

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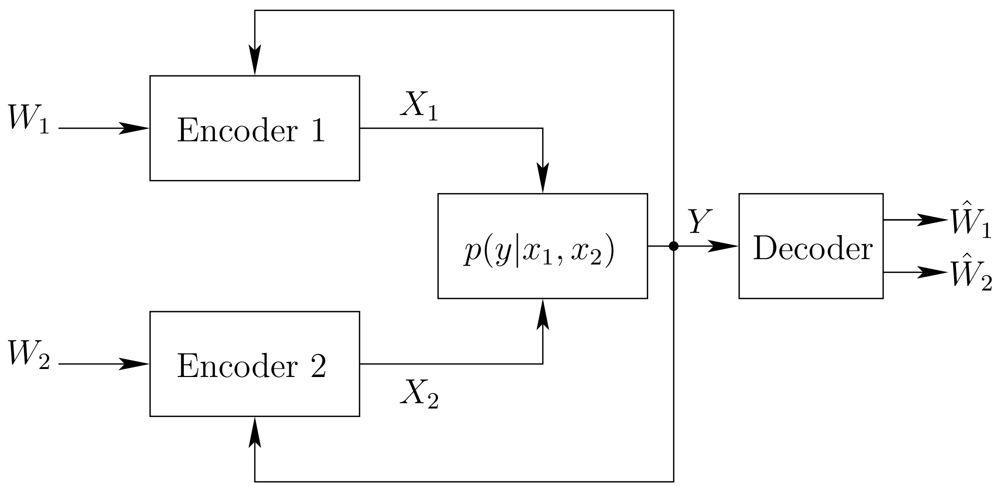
The Multiple Access Channel with Feedback



- Feedback increases capacity for MAC [Gaarder, Wolf].
- Shown for the binary erasure MAC, where $Y = X_1 + X_2$.
- Cover-Leung achievable rate region is the feedback capacity region, if $X_1 = f(X_2, Y)$ [Willems].
- Capacity region is not known in general.

Cut-set Outer Bound for MAC-FB

- Cut-set outer bound:

$$\mathcal{CS} = \Big\{(R_1, R_2) : R_1 \leq I(X_1; Y|X_2) \\ R_2 \leq I(X_2; Y|X_1) \\ R_1 + R_2 \leq I(X_1, X_2; Y)\Big\}$$

  where the random variables (X_1, X_2, Y) have the joint distribution

$$p(x_1, x_2, y) = p(x_1, x_2)p(y|x_1, x_2)$$

- $\mathcal{CS}$ bound is tight for the two-user Gaussian MAC with feedback [Ozarow], where $Y = X_1 + X_2 + Z$, and $Z \sim \mathcal{N}(0, \sigma^2)$.

- We consider a corresponding discrete, binary version of MAC, $Y = X_1 + X_2 + N$, and X_1, X_2, N are all binary with N being uniform on $\{0, 1\}$.

New Outer Bounds based on Dependence Balance

- New outer bounds:

$$\mathcal{DB}_{PC}^{(1)} = \Big\{(R_1,R_2) : R_1 \leq I(X_1;Y|X_2,T) + H(X_1|Y,X_2,T)$$
$$R_2 \leq I(X_2;Y|X_1,T)$$
$$R_1 \leq I(X_1;Y|X_2)$$
$$R_1 + R_2 \leq I(X_1,X_2;Y)\Big\}$$

and

$$\mathcal{DB}_{PC}^{(2)} = \Big\{(R_1,R_2) : R_1 \leq I(X_1;Y|X_2,T)$$
$$R_2 \leq I(X_2;Y|X_1,T) + H(X_2|Y,X_1,T)$$
$$R_2 \leq I(X_2;Y|X_1)$$
$$R_1 + R_2 \leq I(X_1,X_2;Y)\Big\}$$

where (X_1,X_2,Y,T) have a joint distribution

$$p(t,x_1,x_2,y) = p(t)p(x_1|t)p(x_2|t)p(y|x_1,x_2)$$

and T is subject to a cardinality constraint $|\mathcal{T}| \leq |\mathcal{X}_1||\mathcal{X}_2| + 3$.

Binary Additive Noisy MAC-FB

- We consider the following channel

$$Y = X_1 + X_2 + N$$

  where X_1, X_2 and N are binary and N is uniform on $\{0,1\}$.

- This channel does not fall into the class for which $X_1 = f(X_2, Y)$.
- For the symmetric-rate point (R,R) in the feedback capacity region,
  - Cover-Leung inner bound $= 0.43621$ bits/transmission.
  - Kramer's inner bound $= 0.43879$ bits/transmission.
  - Cut-set upper bound $= 0.45915$ bits/transmission.
  - Our upper bound $= 0.45330$ bits/transmission.

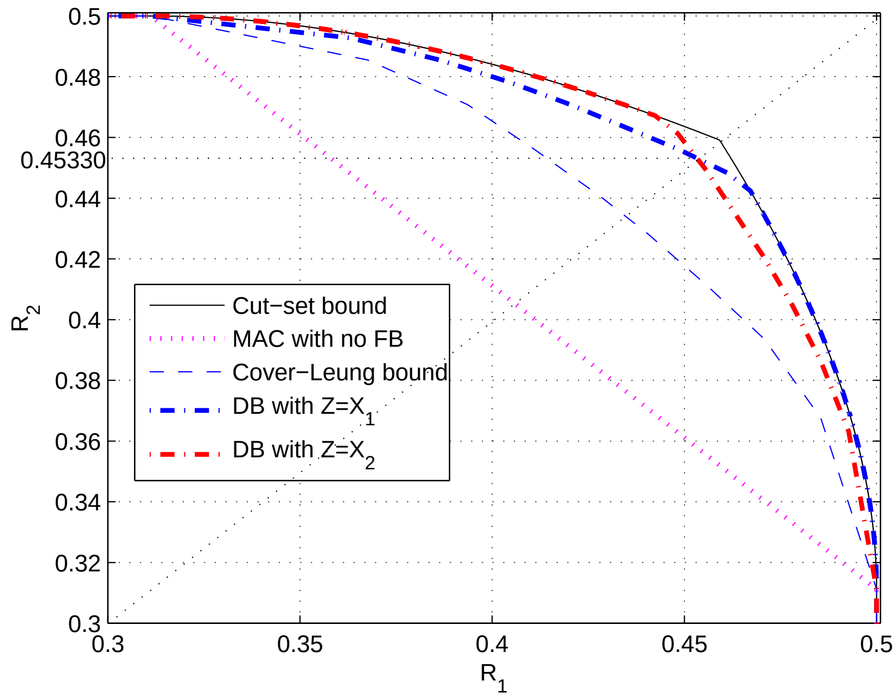
Evaluation of Our Upper Bounds

- Difficult to evaluate: due to auxiliary random variable T.
- For binary X_1, X_2, we have $|\mathcal{T}| \leq 7$.
- We use the following composite function:

$$\phi(s) = \begin{cases} \frac{1-\sqrt{1-2s}}{2}, & \text{for } 0 \leq s \leq 1/2 \\ \frac{1-\sqrt{2s-1}}{2}, & \text{for } 1/2 < s \leq 1 \end{cases}$$

- Using $\phi(s)$ and some of its new properties, we show that $|\mathcal{T}| = 2$ is sufficient to evaluate our upper bound for symmetric-rate point (R, R).
- This approach also used in [Kramer, 2003] and [Willems, 1982] to evaluate the Cover-Leung achievable rate region.

Illustration of Our Bounds and the Cut-set Bound



- Our bounds improve upon the $\mathcal{CS}$ bound for all points where feedback increases capacity.
- In general, the cut-set bound is not tight for the multiple access channel with feedback.