Diversity Order Gain with Noisy Feedback in Multiple Access Channels

Vaneet Aggarwal
Princeton University

Joint Work with Ashutosh Sabharwal, Rice University

June 3, 2008

Outline

- Channel Model
- Introduction to Diversity Multiplexing Tradeoff
- Diversity Multiplexing tradeoff for MIMO
- Diversity Multiplexing tradeoff for MIMO MAC with no feedback - Key features
- What do we gain with quantized feedback

Channel Model

1. m transmit antennas
2. n receive antennas
3. Slow Fading Path gains i.i.d. Rayleigh, change every T seconds

Diversity Multiplexing Tradeoff

1. In wireless communications, we need high data rate and high reliability. Zheng and Tse formulated a basic tradeoff between the two at high SNR.

A diversity gain $d^*(r)$ is achieved at multiplexing gain r if

$$R = r \log \mathsf{SNR}$$

and

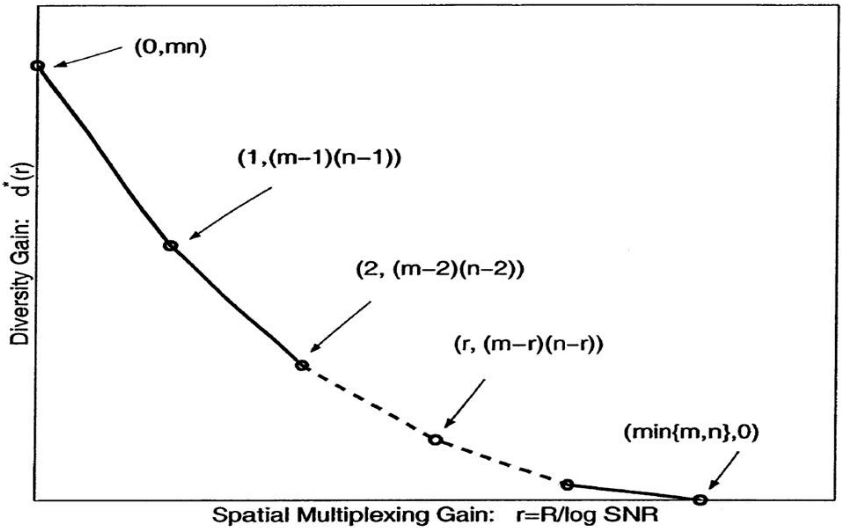
$$p_{\text{out}}(R) \approx \mathsf{SNR}^{-d^*(r)},$$

or more precisely,

$$\lim_{\mathsf{SNR}\to\infty} \frac{\log p_{\text{out}}(r \log \mathsf{SNR})}{\log \mathsf{SNR}} = -d^*(r).$$

The curve $d^*(\cdot)$ is the diversity–multiplexing tradeoff of the slow fading channel.

Diversity Multiplexing Tradeoff for MIMO system



Diversity Multiplexing Tradeoff for MAC system

1. Let r_1 and r_2 be the multiplexing of the two users.
2. The diversity is $\min(G_{m,n}(r_1), G_{1m,n}(r_2), G_{2m,n}(r_1 + r_2))$ [Tse Viswanath Zheng 04].

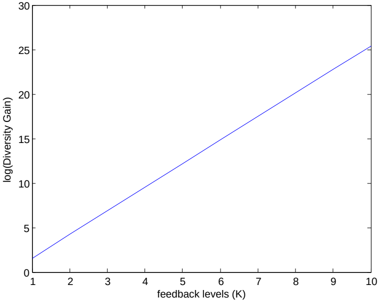
Why feedback?

1. Receiver knows the channel
2. It can send some information to the transmitter.
3. How better do we perform?

Gains with feedback

1. We use the feedback to perform power control at the transmitters.
2. Power control scheme is simple - use K power levels and select the minimum by which we get no outage in case we can avoid it. Then, optimize on the power levels.
3. Maximum Diversity for $\log(K)$ bits feedback $= mn\frac{(mn)^K - 1}{mn-1}$ [Aggarwal Sabharwal ISIT'08]
4. Diversity increases exponentially with K
5. Note that as bits were $\log(K)$, diversity increases double exponentially with feedback.

Gains with feedback, Example $m = 2$, $n = 7$,$r_1 = 1$, $r_2 = 1.2$



Did we miss something?

1. The feedback channel can never be perfect. There will always be errors.
2. Let us assume reverse channel to be SNR-symmetric and hence has the error of the order of SNR^{-mn}.
3. What do we get?

Noisy feedback

1. The achievable diversity $2mn$, which can be achieved for just 1 bit of feedback. This is also the maximum diversity for MIMO system for the above mentioned power control policy. [Aggarwal Sabharwal, ISIT 2008]
2. The gain at any multiplexing pair do not increase beyond mn.

Noisy feedback (Contd.)

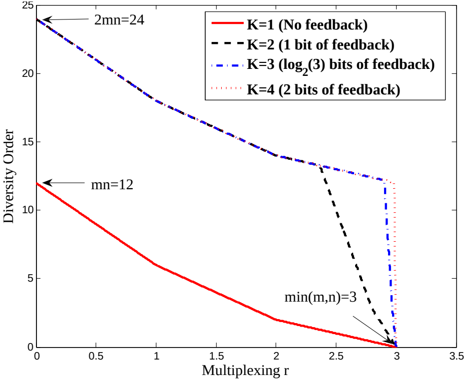


Figure: Diversity-Multiplexing Tradeoff for MIMO Channel ($m = 3, n = 4$).

Noisy feedback (Contd.)

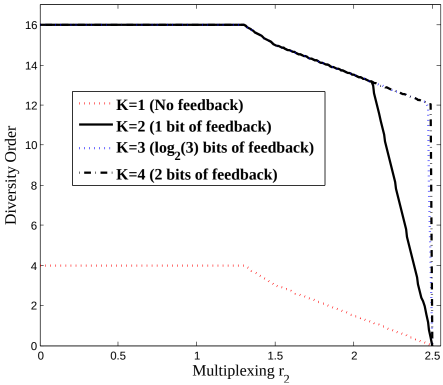


Figure: Diversity-Multiplexing Tradeoff for MAC ($m = 3, n = 4,\ r_1 = 1.5$).

Conclusions

1. There is super-exponential gain in diversity with bits of feedback if we assume that the feedback is perfect.
2. If the feedback link has noise, the gains are bounded.
3. We can get double maximum diversity with just a single bit of feedback.
4. At any multiplexing, we can gain diversity of mn, albeit requiring more than 1 bit feedback.